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ORIGINAL ARTICLE

A Centroid-based Ranking Method of Trapezoidal Intuitionistic Fuzzy Numbers and Its Application to MCDM Problems

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Abstract The objective of this paper is to introduce a novel method to compare trapezoidal intuitionistic fuzzy numbers (TrIFNs). Till now little research has been done regarding the ranking of TrIFNs. This paper first reviews the existing ranking methods and shows their drawbacks by using several examples. In order to overcome the drawbacks of the existing methods, a new ranking method of TrIFNs is developed by utilizing the concept of centroid point. For this purpose, centroid point for TrIFN is also defined. The rationality validation of the proposed centroid formulae is proved. Further, the ranking method is applied to a multi-criteria decision making (MCDM) problem in which the ratings of the alternatives on criteria are expressed with TrIFNs. Finally, the effectiveness and applicability of the proposed ranking method are illustrated with an aerospace research organization center selection example. This article has also justified the proposed approach by analyzing a comparative study.

Keywords Intuitionistic fuzzy number · Ranking · Centroid point · Multi-criteria decision making

1. Introduction

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An important generalization of classical fuzzy set theory [1] is the theory of intuitionistic fuzzy set (IFS), introduced by Atanassov [2]. Fuzzy set, projected as a framework for modeling uncertainty, assigns to each element of the universe of discourse a degree of membership between zero and one. The degree of non-membership is considered as complement to one of the membership degree. But IFS implicates the fact that the non-membership degree is not always complement to the membership degree. There may arise some hesitation degree and these membership, non-membership and hesitation degrees to an alternative can be suitably modeled by intuitionistic fuzzy values (IFVs) (which are the basic components of IFSs [3]). Thus, there are some situations where IFS theory is more suitable to deal with incomplete or inexact information present in real-world applications. Eventually, in less than three decades since its first appearance, IFS theory has been investigated by many authors and applied in different areas including decision making. Now in modeling a decision making problem, ranking is an important issue. In this regard, various ranking methods of IFSs have been proposed and used for decision making problems [3-16]. Nevertheless, the research concentrated on the finite universe of discourse only. In view of this, from the concept of IFSs, intuitionistic fuzzy numbers (IFNs) have been defined [17-22] with the universe of discourse as the real line. The domains of IFSs are discrete universe of discourses. The concept of IFNs can be viewed as an extensive approach to define an IFS in the case when available information is not sufficient enough to define a conventional IFS based on discrete sets. Moreover, in information integration process, discrete sets may loss partial information [23-26] while continuous sets maintain the integrity of information and, thus, are more capable to model incomplete and abundant information than discrete sets. With this view, extending the concept of discrete sets to continuous sets, IFNs have been defined which can more suitably model imprecise data involved in real-world decision making problems.

In recent times, IFNs have received increasing [22, 26-31] attention because of their ability to handle imprecise and abundant information in decision making situation. The aim of the present study is not to cover all the range of IFNs but merely to address the ranking of IFNs and its application to MCDM problems. Now, the traditional ranking methods for comparing IFSs cannot deal with IFNs, as the former is based on discrete sets and later on continuous sets. In this view, to rank and compare IFNs, several ranking methods have been proposed. In 2008, Nayagam et al. [32] introduced a new score function for ranking triangular IFNs (TIFNs) and further they modified it in [33]. Wang and Zhong [34] used both the score and accuracy functions to rank TrIFNs and developed an MCDM problem based on the proposed ranking process. By calculating normalized Hamming distances from IFNs to positive and negative ideal solutions, a ranking method was provided by Wei [35] and subsequently a group decision making problem was also investigated. Wei and Tang [36] proposed a method for ranking IFNs by utilizing possibility degree and applied it for solving an MCDM problem. A new ranking method was developed by Li [37] on the basis of the idea of a ratio to the value index and ambiguity index of TIFNs. By utilizing the same value index and ambiguity index, Dubey and Mehra [38] proposed a new ranking function to rank IFNs. Rezvani [39] also proposed a ranking process of TrIFNs by determining value and ambiguity of TrIFNs.

However, after analyzing the aforementioned ranking procedures it is observed that, in some cases, they fail to calculate the ranking results correctly. Also, many of them compute different ranking results for the same problem. Under these circumstances, the decision maker may not be able to carry out the comparison and recognition properly. This creates a problem in practical applications. In order to overcome the shortcomings of the existing methods, a new method for ranking TrIFNs is proposed in this paper which is based on centroid point of TrIFNs.

This paper is planned as follows: In Section 2, basic ideas of IFSs, IFVs, TrIFNs and a brief review of the existing ranking methods of IFVs are introduced. A brief description of the existing ranking procedures of TrIFNs/TIFNs is given in Section 3. Section 4 represents the centroid formulae based ranking algorithm of TrIFNs. In this section, rationality of the centroid formulae is justified. A set of examples is provided to compare the proposed ranking method with the existing ranking methods of TrIFNs/TIFNs. An application of the proposed ranking process in MCDM problem under intuitionistic fuzzy environment is analyzed in Section 5. In Section 6, a numerical example is used to illustrate the proposed method and the comparison analysis is also conducted. The paper is concluded in Section 7.

2. Preliminaries

Here we present definitions and concepts that will be required for our subsequent developments.

2.1. Intuitionistic Fuzzy Set

Definition 2.1 [2] *An IFS $\widetilde{A}$ in the universe X can be expressed in a set of ordered triple, $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x), \nu_{\widetilde{A}}(x)) \mid x \in X\}$, where $\mu_{\widetilde{A}}(x) \mid X \to [0,1]$ is the degree of belongingness and $\nu_{\widetilde{A}}(x) \mid X \to [0,1]$ is the degree of non-belongingness of x in $\widetilde{A}$. They satisfy the relation $0 \leq \mu_{\widetilde{A}}(x) + \nu_{\widetilde{A}}(x) \leq 1$ $\forall x \in X$. Each fuzzy set is a special case of IFS and it can be represented as $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x), 1 - \mu_{\widetilde{A}}(x)) \mid x \in X\}$. The quantity $\pi_{\widetilde{A}}(x) = 1 - \mu_{\widetilde{A}}(x) - \nu_{\widetilde{A}}(x)$ is called the degree of hesitation (indeterminacy) of x in $\widetilde{A}$. When $\pi_{\widetilde{A}}(x) = 0$ $\forall x \in X$, IFS becomes fuzzy set.*

For convenience of computation, the 2-tuple $\widetilde{\alpha} = (\mu_{\widetilde{\alpha}}, \nu_{\widetilde{\alpha}})$ is called an IFV [12] where $\mu_{\widetilde{\alpha}} \in [0,1]$, $\nu_{\widetilde{\alpha}} \in [0,1]$ and $\mu_{\widetilde{\alpha}} \leq 1 - \nu_{\widetilde{\alpha}}$. Subsequently, $\pi_{\widetilde{\alpha}} = 1 - \mu_{\widetilde{\alpha}} - \nu_{\widetilde{\alpha}}$ is the hesitation margin of IFV $\widetilde{\alpha}$.

2.2. Representing Experts' Opinions by Using TrIFNs

Experts always feel comfortable to express their opinions in natural languages. So the first task is to quantify these linguistic expressions. In real-world decision making problems, in the situation of imprecise or vague information, an expert's satisfaction and dissatisfaction, regarding his (her) opinion may not be complement to each other. In addition, due to abundance of information, maintaining integrity in information processing is another important aspect. With this view, to capture satisfaction and dissatisfaction and to maintain the continuity of information, quantifying experts' opinion using IFNs is more suitable than IFSs [21-23, 28, 37]. A brief description of the concept of TrIFNs is given below.

Definition 2.2 [40] *A TrIFN $A = \langle [(a, b, c, d), w;], [(a', b, c, d'), u] \rangle$ is a special IFV on the real line $\mathbb{R}$ and its membership and non-membership functions are respectively defined as follows:*

$$\mu_A(x) = \begin{cases} \dfrac{(x-a)w}{(b-a)}, & \text{for } a \le x \le b, \\ w, & \text{for } b \le x \le c, \\ \dfrac{(d-x)w}{(d-c)}, & \text{for } c \le x \le d, \\ 0, & \text{for } x \le a, x \ge d \end{cases} \tag{1}$$

and

$$\nu_A(x) = \begin{cases} \dfrac{(b-x)+(x-a')u}{(b-a')}, & \text{for } a' \le x \le b, \\ u, & \text{for } b \le x \le c, \\ \dfrac{(x-c)+(d'-x)u}{(d'-c)}, & \text{for } c \le x \le d', \\ 1, & \text{for } x \le a', x \ge d', \end{cases} \tag{2}$$

where w and u represent the maximum degree of membership and the minimum degree of non-membership of an element $x \in A$, respectively, such that they satisfy the conditions: $0 \le w, u \le 1$; $0 \le w + u \le 1$. The function $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called hesitancy or indeterminacy of an element x belonging to A.

For the sake of simplicity in computation and without any loss of generality, throughout this paper we have considered $a = a'$ and $d = d'$. Symbolically, then TrIFN can be represented as $A = [(a, b, c, d); w, u]$ (Fig.7). If $b = c$, then TrIFN transforms to a TIFN, i.e., TIFN is a special case of TrIFN. It is to be noted that 'TrIFN' defined in Definition 2.2, can be viewed as an extensive mathematical expression of the IFN of Xu and Yager [12, 13].

Remark 2.1 If TrIFN $A = [(a, b, c, d); w, u]$ represents an expert's opinion, then the parameters w and u denote the corresponding degrees of satisfaction and dissatisfaction of the expert, respectively. In the context of MCDM, TrIFN A allows one to simulate human cognitive processes, viz., thinking, reasoning etc. in a more suitable manner than IFVs representation. In many practical cases, it is also observed that a decision maker may not be able to get proper decision results based on the uncertain/imprecise information quantified by IFVs. In this situation, remodeling available decision information is required and TrIFNs provide a more appropriate tool to model such uncertain information. For example, TrIFN $A = [(a, b, c, d); w, u]$ may represent an imprecise quantity 'approximately A', which is approximately equal to A. The imprecise quantity 'approximately A' is expressed using any value between a and d with different degrees of satisfaction and dissatisfaction of the expert. In other words, the most possible value is occurred in between b and c with the satisfaction level w and the dissatisfaction level u; the pessimistic value is a with the satisfaction level 0 and the dissatisfaction level 1; the optimistic value is d with the satisfaction level 0 and the dissatisfaction level 1; other values are in (a, d) with satisfaction degree $\mu_A(x)$ and the dissatisfaction degree $\nu_A(x)$ defined in Equations (1) and (2).

We now recall the definitions and some useful operations involving TrIFNs.

Definition 2.3 [28] *Let $A = [(a, b, c, d); w, u]$ be a TrIFN. If $a \geq 0$ and one of the four values a, b, c and d is not equal to 0, then the TrIFN A is called positive TrIFN.*

Definition 2.4 [37] *Let $A_1 = [(a_1, b_1, c_1, d_1); w_1, u_1]$ and $A_2 = [(a_2, b_2, c_2, d_2); w_2, u_2]$ be two positive TrIFNs and $K \geq 0$ be a scalar. Then*

(1) $A_1 \oplus A_2 = [(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); \min(w_1, w_2), \max(u_1, u_2)]$.

(2) $A_1 \otimes A_2 = [(a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2); \min(w_1, w_2), \max(u_1, u_2)]$.

(3) $KA_1 = [(Ka_1, Kb_1, Kc_1, Kd_1); w_1, u_1]$.

(4) $A_1^K = [(a_1^K, b_1^K, c_1^K, d_1^K); w_1, u_1]$.

Definition 2.5 *Let $A = [(a, b, c, d); w, u]$ be a TrIFN. The support of TrIFN A is defined as support$(A) = d - a$.*

Definition 2.6 [37] *Let $A_i = [(a_i, b_i, c_i, d_i); w_i, u_i]$ $(i = 1, 2, \cdots, n)$ be a set of TrIFNs. Then the trapezoidal intuitionistic fuzzy weighted arithmetic mean (TrIFWAM) operator is defined as follows:*

$$\begin{aligned} &TrIFWAM(A_1, A_2, \cdots, A_n) \\ &= \Big[\Big(\sum_{i=1}^{n} t_i a_i, \sum_{i=1}^{n} t_i b_i, \sum_{i=1}^{n} t_i c_i, \sum_{i=1}^{n} t_i d_i\Big); \min_i\{w_i\}, \max_i\{u_i\}\Big], \end{aligned} \tag{3}$$

where $t = (t_1, t_2, \cdots, t_n)^T$ is the weight vector of A_i, $t_i \in [0, 1]$ and $\sum_{i=1}^{n} t_i = 1$.

2.3. Existing Ranking Methods of IFVs

There are many papers discussing various ranking methods of IFVs, (or IFSs) and for a recent review we refer the reader to [14, 41], see also recent papers [3-6]. Here, we present an overview of the existing ranking methods to IFVs. A particular review is based on counterintuitive examples, that involve two distinct sets having the same ranking values, i.e., appearing to have incomparable. For examples, we cite [3, 9, 12-15, 42].

As an illustration, Chen and Tan [9] considered the score function to rank IFVs $\widetilde{\alpha}_i$ which is defined as follows:

$$S(\widetilde{\alpha}_i) = \mu_{\widetilde{\alpha}_i} - \nu_{\widetilde{\alpha}_i}.$$

The larger the score value, the greater the IFV.

Hong and Choi [42] found that score function based ranking method shown unreasonable results and they presented the following function, known as accuracy function of IFVs,

$$H(\widetilde{\alpha}_i) = \mu_{\widetilde{\alpha}_i} + \nu_{\widetilde{\alpha}_i}.$$

The larger the accuracy value, the greater the IFV. However, ranking is not possible by using an accuracy function if two IFVs have the same accuracy values.

Xu and Yager [12] tried to overcome the drawback raised in standalone score function and accuracy function based ranking methods, by providing a more realistic strong ranking process, using both of them in the following way:

(a) If $S(\widetilde{\alpha}_i) > S(\widetilde{\alpha}_j)$, then $\widetilde{\alpha}_i > \widetilde{\alpha}_j$;

(b) If $S(\widetilde{\alpha}_i) = S(\widetilde{\alpha}_j)$, then
if $H(\widetilde{\alpha}_i) > H(\widetilde{\alpha}_j)$, then $\widetilde{\alpha}_i > \widetilde{\alpha}_j$;
$H(\widetilde{\alpha}_i) = H(\widetilde{\alpha}_j)$, then $\widetilde{\alpha}_i = \widetilde{\alpha}_j$.

In the meantime, few authors have tried to develop the comparison method by means of the intuitionistic fuzzy distance measures instead of score and accuracy functions. Xu and Yager [13] compared IFVs by computing distance between each IFV $\widetilde{\alpha}_i$ and the positive ideal point $\widetilde{\alpha}^+ = (1, 0)$ in the following way.

$$d_1(\widetilde{\alpha}_i, \widetilde{\alpha}^+) = \frac{1}{2}(|\mu_{\widetilde{\alpha}_i} - 1| + |\nu_{\widetilde{\alpha}_i} - 0|).$$

The smaller the distance value, the bigger the IFV. By analyzing above distance based ranking process, it is found that this distance has connection with the score function $\left(\text{as } d_1(\widetilde{\alpha}_i, \widetilde{\alpha}^+) = 1 - \frac{S(\widetilde{\alpha}_i)}{2}\right)$ and consequently, it has the limitation of the score function. To overcome this drawback, Szmidt and Kacprzyk [43] improved this distance measure by taking into account the hesitation part, which is defined as follows:

$$d_2(\widetilde{\alpha}_i, \widetilde{\alpha}^+) = \frac{1}{2}(|\mu_{\widetilde{\alpha}_i} - 1| + |\nu_{\widetilde{\alpha}_i} - 0| + |\pi_{\widetilde{\alpha}_i} - 0|).$$

Again, $d_2(\widetilde{\alpha}_i, \widetilde{\alpha}_i^+) = 1 - \mu_{\widetilde{\alpha}_i}$, i.e., this distance only relies on the membership value of $\widetilde{\alpha}_i$, so it losses some useful information [14]. Keeping in mind the imperfectness of these distances, to compare IFVs, Xu [15] used a ranking function which is defined as follows:

$$R_1(\widetilde{\alpha}_i) = \frac{1}{2}(1 + \pi_{\widetilde{\alpha}_i})d_2(\widetilde{\alpha}_i, \widetilde{\alpha}^+).$$

The smaller the R_1 value, the larger the IFV. However, Yu et al. [14] showed that, the comparison method based on the above ranking function R_1 cannot satisfy the essential properties of the operational laws of ranking. In an attempt to resolve such a problem, they developed a new ranking index based on the dominance relation of

IFVs. The ranking function based on the dominance matrix is given as follows:

$$R_2(\widetilde{\alpha}_i) = \frac{1}{n(n-1)}\Big(\sum_{j=1}^{n} d_{ij} + \frac{n}{2} - 1\Big).$$

Ranking is done according to greater R_2 values. Here, $D = (d_{ij})_{n\times n}$ is said to be a dominance matrix if $\widetilde{\alpha}_i \geq \widetilde{\alpha}_j$ and $d_{ij} = d_{\widetilde{\alpha}_i\widetilde{\alpha}_j}$ can be computed as

$$d_{\widetilde{\alpha}_i\widetilde{\alpha}_j} = \begin{cases} 1, & \text{for } \mu_{\widetilde{\alpha}_i} \geq \mu_{\widetilde{\alpha}_j},\ \ \nu_{\widetilde{\alpha}_i} \leq \nu_{\widetilde{\alpha}_j}, \\ \dfrac{(\mu_{\widetilde{\alpha}_i} - \mu_{\widetilde{\alpha}_j})\nu_{\widetilde{\alpha}_i}}{2(1-\mu_{\widetilde{\alpha}_j})\nu_{\widetilde{\alpha}_i} - (1-\mu_{\widetilde{\alpha}_i})\nu_{\widetilde{\alpha}_i} - (1-\mu_{\widetilde{\alpha}_j})\nu_{\widetilde{\alpha}_j}}, & \text{for otherwise}, \end{cases}$$

and $d_{ii} = 0.5$, $d_{ji} = 1 - d_{ij}\ \forall i, j$. Furthermore, this dominance matrix has identical form of a fuzzy preference relation [16] as each element d_{ij} in the dominance matrix D represents the possibility that $\widetilde{\alpha}_i$ is larger than $\widetilde{\alpha}_j$.

Here, an example is considered to show the drawback of this dominance relation based ranking method of IFVs.

Example 2.1 Let $\widetilde{\alpha}_1 = (0.4, 0.3)$ and $\widetilde{\alpha}_2 = (0.55, 0.4)$ be two IFVs. The ranking result by Yu et al.'s [14] method is $R_2(\widetilde{\alpha}_1) = 0.5 = R_2(\widetilde{\alpha}_2)$. This implies that $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ are not comparable. It is to be noted that $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ are not identical, so there should be a ranking order between them. Hence, in some cases, this method does not provide a satisfactory result.

Moreover, motivated by one of the prominent decision making technique TOPSIS (the technique for order of preference by similarity to ideal solution), two ranking methods for IFVs were developed. As an illustration, to compare and rank n IFVs $\widetilde{\alpha}_i$, Xu and Yager [13] utilized the following one:

$$C(\widetilde{\alpha}_i) = \frac{d_2(\widetilde{\alpha}_i, \widetilde{\alpha}_i^-)}{d_2(\widetilde{\alpha}_i, \widetilde{\alpha}_i^-) + d_2(\widetilde{\alpha}_i, \widetilde{\alpha}_i^+)},$$

where $d_2(\widetilde{\alpha}_i, \widetilde{\alpha}^-)$ and $d_2(\widetilde{\alpha}_i, \widetilde{\alpha}^+)$ are Hamming distances [43] of $\widetilde{\alpha}_i$ from negative ideal solution $\widetilde{\alpha}^- = (0, 1)$ and positive ideal solution $\widetilde{\alpha}^+ = (1, 0)$, respectively. Now, ranking is done according to $C(\widetilde{\alpha}_i)$ $(i = 1, 2, \cdots, n)$ values, the greater the value $C(\widetilde{\alpha}_i)$, the better the alternative $\widetilde{\alpha}_i$.

Furthermore, Zhang and Xu [3], compared IFVs according to similarity and accuracy values. The similarity index for IFV $\widetilde{\alpha}_i$ is computed based on TOPSIS index as follows:

$$L(\widetilde{\alpha}_i) = 1 - \frac{d_2(\widetilde{\alpha}_i, \widetilde{\alpha}^+)}{d_2(\widetilde{\alpha}_i, \widetilde{\alpha}^-) + d_2(\widetilde{\alpha}_i, \widetilde{\alpha}^+)}.$$

Ranking is done according to the order of L values, IFV with larger L value should be ranked first. If for two IFVs, L values are the same, then ranking is done according to greater accuracy values.

Here, an example is considered to show the drawback of the TOPSIS based ranking indices.

Example 2.2 For two IFVs $\widetilde{\alpha}_1 = (0.6, 0.2)$ and $\widetilde{\alpha}_2 = (0.5, 0)$, the ranking result by Xu and Yager's [13] method and Zhang and Xu's [3] method is $C(\widetilde{\alpha}_1) = \frac{2}{3} = C(\widetilde{\alpha}_2)$ and $L(\widetilde{\alpha}_1) = \frac{2}{3} = L(\widetilde{\alpha}_2)$. This implies that $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ are not comparable although $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ are not identical. So, there should be a ranking order between them. Hence, TOPSIS based ranking indices fail to provide a satisfactory ranking result in some cases.

A set of example of IFVs is constructed for giving a comparative analysis among the existing ranking measures and the results are shown in the following Table 1.

Table 1: A comparison result of the existing methods for IFVs.

Example set	Existing methods	Ranking results
$\widetilde{\alpha}_1 = (0.4, 0.4)$	Chen and Tan [9]	$\widetilde{\alpha}_3 \succ \widetilde{\alpha}_2 \succ \widetilde{\alpha}_1$
$\widetilde{\alpha}_2 = (0.5, 0.4)$	Hong and Choi [42]	$\widetilde{\alpha}_2 \succ \widetilde{\alpha}_1 \succ \widetilde{\alpha}_3$
$\widetilde{\alpha}_3 = (0.6, 0.1)$	Xu and Yager [12]	$\widetilde{\alpha}_3 \succ \widetilde{\alpha}_2 \succ \widetilde{\alpha}_1$
	Szmidt and Kacprzyk [43]	$\widetilde{\alpha}_3 \succ \widetilde{\alpha}_2 \succ \widetilde{\alpha}_1$
	Xu [15]	$\widetilde{\alpha}_3 \succ \widetilde{\alpha}_2 \succ \widetilde{\alpha}_1$
	Yu et al. [14]	$\widetilde{\alpha}_3 \succ \widetilde{\alpha}_1 \succ \widetilde{\alpha}_2$
	Xu and Yager [13]	$\widetilde{\alpha}_3 \succ \widetilde{\alpha}_2 \succ \widetilde{\alpha}_1$
	Zhang and Xu [3]	$\widetilde{\alpha}_3 \succ \widetilde{\alpha}_2 \succ \widetilde{\alpha}_1$

In Table 1, an example set with three IFVs $\widetilde{\alpha}_1 = (0.4, 0.4)$, $\widetilde{\alpha}_2 = (0.5, 0.4)$ and $\widetilde{\alpha}_3 = (0.6, 0.1)$ is considered. From the above example set, it is observed that, $\widetilde{\alpha}_3$ should be better than $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ as $\widetilde{\alpha}_3$ has greater membership value as well as smaller non-membership value. Again, $\widetilde{\alpha}_2$ is better than $\widetilde{\alpha}_1$ as $\widetilde{\alpha}_2$ has greater membership value than $\widetilde{\alpha}_1$, while they have equal non-membership values. Hence, from the human intuition, ranking order should be $\widetilde{\alpha}_3 \succ \widetilde{\alpha}_2 \succ \widetilde{\alpha}_1$. This result matches with the ranking results obtained by Chen and Tan's [9], Xu and Yager's [12], Szmidt and Kacprzyk's [43], Xu's [15], Xu and Yager's [13] and Zhang and Xu's [3] ranking processes. Furthermore, the best number, $\widetilde{\alpha}_3$, obtained through Yu et al.'s [14] method, is the same as the result obtained through most of the ranking processes, but the ranking order of other two numbers, obtained by Yu et al.'s [14] method, is somewhat different from the ranking order presented through the others. However, ranking order obtained by Hong and Choi's [42] process is complectly different from the others as it provides the result by summing membership and non-membership values.

Remark 2.2 To deal with practical decision problems under intuitionistic fuzzy environment, the above research works provide a rich contain to the theory of IFVs. In our study, we consider TrIFN which is a special type of IFV defined on the real line $\mathbb{R}$, i.e., on continuous universe of discourse. Thus, the traditional ranking methods to IFVs can not be compared with TrIFNs. In order to provide more clarity, we sincerely tried to extend the existing ranking methods of IFVs to rank TrIFNs. Even though we have sincerely tried, we are not able to get satisfactory results. The discussion is provided below.

In order to compare TrIFNS, first we make an attempt to extend the ranking approach of Xu and Yager [13] towards the continuous universe. Motivated by the mentioned research work [13], a ranking index for each TrIFN $A_i = [(a_i, b_i, c_i, d_i); w_i, u_i]$ can be defined as follows:

$$C(A_i) = \frac{d(A_i, R^-)}{d(A_i, R^-) + d(A_i, R^+)},$$

where $d(A_i, R^-)$ and $d(A_i, R^+)$ are Hamming distances [35] of TrIFN A_i from negative ideal solution (NIS) $R^- = [(0, 0, 0, 0); 0, 1]$ and positive ideal solution (PIS) $R^+ = [(1, 1, 1, 1); 1, 0]$, respectively. Ranking is done according to greater closeness coefficient values.

Here an example is considered to show the limitation of this approach.

Example 2.3 For two TrIFNs $A_1 = [(0.57, 0.73, 0.73, 0.83); 0.73, 0.2]$ and $A_2 = [(0.58, 0.74, 0.74, 0.819); 0.72, 0.2]$, according to the above formula, we get $C(A_1) = 0.547 = C(A_2)$. This implies that A_1 and A_2 are not comparable. It is to be noted that, A_1 and A_2 are not identical, so there should be a ranking order between them. Hence, the result obtained by this extended ranking method is not satisfactory.

The extension of score and accuracy functions based ranking method [12] of IFVs, towards the continuous universe (i.e., score and accuracy functions based ranking method of TrIFNs) is proposed by Wang and Zhong [34]. We will analyze this method in Section 3.3 (during analysis of existing ranking methods of TrIFNs).

In paper [3], ranking of IFVs is done according to TOPSIS based ranking index. Inspired by the mentioned research work [3], a new ranking index for TrIFN $A_i = [(a_i, b_i, c_i, d_i); w_i, u_i]$ can be defined as

$$L(A_i) = 1 - \frac{d(A_i, R^+)}{d(A_i, R^-) + d(A_i, R^+)}.$$

Ranking is done according to the order of L values, TrIFN with larger L value should be ranked first. If for two TrIFNs, L values are the same, then ranking is done according to greater accuracy values. The accuracy value for TrIFN is defined in [34].

An example is considered to show the drawback of the above ranking method.

Example 2.4 For two TrIFNs $A_1 = [(0.56, 0.74, 0.8, 0.9); 0.5, 0.5]$ and $A_2 = [(0.5, 0.7, 0.85, 0.95); 0.5, 0.5]$, according to the above formula, we get the result as $L(A_1) = 0.375 = L(A_2)$ and $H(A_1) = 0.375 = H(A_2)$. This implies that A_1 and A_2 are not comparable, although A_1 and A_2 are not identical, so there should be a ranking order between them. Hence, the result obtained by this extended ranking method is not satisfactory.

The foregoing discussion shows that, the extension of existing ranking methods to IFVs towards the continuous universe of discourse, fail to provide satisfactory results in some cases. Now, we will focus our attention on the analysis of existing ranking methods defined in continuous universe of discourse [34, 35, 37-39] to rank TrIFNs. Additionally, we will provide counter intuitive examples to show their drawbacks.

3. A Brief Note on the Existing Ranking Methods of TrIFNs

As mentioned in the introduction, ranking of TrIFNs is often a necessary step in solving the problems under intuitionistic fuzzy environment and literature review reckoned the existence of five to six ranking methods. In this section, almost all the existing ranking processes of TrIFNs are analyzed and their drawbacks/limitations in the computation are pointed out through numerical examples. From literature, it is observed that to rank TrIFNs, existing ranking methods employ some reference points and these reference points are computed by using certain transformation functions, such as, $f : F \to \mathbb{R}$, where F denotes the set of TrIFNs and $\mathbb{R}$ denotes the set of real numbers. By using these transformation functions, TrIFNs are mapped to the reference points that are basically real numbers. Then for any two TrIFNs $A_1, A_2 \in F$, ranking is done according to the value with respect to these reference points on the real line.

Based on the reference points in calculating the ranking index, the existing methods can be classified into following three major categories. We first provide a list of notations which we will use further to describe the existing ranking processes.

List of notations	
R^+	Positive ideal solution for TrIFNs
R^-	Negative ideal solution for TrIFNs
$d(A_i, A_j)$	Distance between TrIFNs A_i and A_j
$v(A_i)$	Value index of TrIFN A_i
$a(A_i)$	Ambiguity index of TrIFN A_i
$R(A_i)$	Ranking index of TrIFN A_i
$s(A_i)$	Score function of TrIFN A_i
$h(A_i)$	Accuracy function of TrIFN A_i
$I(A_i)$	Expected value of TrIFN A_i
$\prec, \approx, \succ$	Ordering relations

3.1. Ideal Solution-based Approach

The notion of ideal solutions is generally used in MCDM problems in which PIS (or NIS) consists of all the best (or worst) values possible for each criterion. With these two reference points PIS and NIS, the basic principle adopted in MCDM is that the chosen alternative should have the shortest distance from the PIS and the farthest distance from the NIS [4, 44-46]. The ideal solution based concept of MCDM was further employed in the development of several methods for ranking fuzzy numbers, for instance, refer to [47-49] and the references therein. Consequently, in 2010, Wei [35] used the concept of ideal solution to comparing and ranking IFNs.

3.1.1. A Review on Wei's Process

In his approach, to compare and rank n TrIFNs $A_i = [(a_i, b_i, c_i, d_i); w_i, u_i]$ $(i = 1, 2, \cdots, n)$, PIS is defined as $R^+ = [(a, b, c, d); w, u] = [(1, 1, 1, 1); 1, 0]$. Then with the ideal solution R^+, defined as above, the distance between each A_i and R^+ is calculated by using the following formula

$$d(A_i, R^+) = \frac{1}{8}\Big[|(1 + w_i - u_i)a_i - (1 + w - u)a| + |(1 + w_i - u_i)b_i - (1 + w - u)b| \\ +|(1 + w_i - u_i)c_i - (1 + w - u)c| + |(1 + w_i - u_i)d_i - (1 + w - u)d|\Big].$$

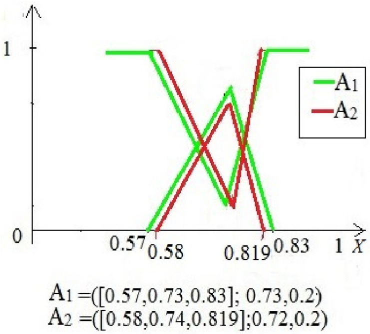

Fig. 1 Example 3.1

Finally, n TrIFNs A_i are compared and ranked according to their distances from R^+, e.g., for any two TrIFNs A_1 and A_2, if $d(A_1, R^+) < d(A_2, R^+)$, then $A_1 > A_2$.

Here an example is demonstrated to show that, for some cases the aforementioned process is unable to give a suitable result.

Example 3.1 For two TIFNs $A_1 = [(0.57, 0.73, 0.83); 0.73, 0.2]$ and $A_2 = [(0.58, 0.74, 0.819); 0.72, 0.2]$, according to Wei's method [35], the distances from R^+ are $d(A_1, R^+) = 0.453 = d(A_2, R^+)$. This implies that A_1 and A_2 (Fig.1) are not comparable. It is to be noted that, A_1 and A_2 are not identical, so there should be a ranking order between them. Hence, the result obtained by Wei's method is not satisfactory.

3.2. Value and Ambiguity Index Based Approach

Delgado et al. [50, 51] have extensively studied two attributes of fuzzy numbers, namely, value and ambiguity, to realize the magnitude and imprecision involved in a fuzzy number. The notion of value and ambiguity appeared to be good parameters to be used together to develop ranking procedures of fuzzy numbers [50-53]. In recent times, the concept of value and ambiguity are also utilized to compare TrIFNs/TIFNs. The methods are described below.

3.2.1. A Review on Li's Process

In order to compare and rank n TIFNs $A_i = [(a_i, b_i, c_i); w_i, u_i]$ $(i = 1, 2, \cdots, n)$, based on the concept of value and ambiguity indices, a ranking index [37] of each TIFN is determined as: $R(A_i) = \frac{v(A_i)}{1+a(A_i)}$, where $v(A_i) = v_\mu(A_i) + \lambda(v_\nu(A_i) - v_\mu(A_i))$ and $a(A_i) = a_\nu(A_i) - \lambda(a_\nu(A_i) - a_\mu(A_i))$.

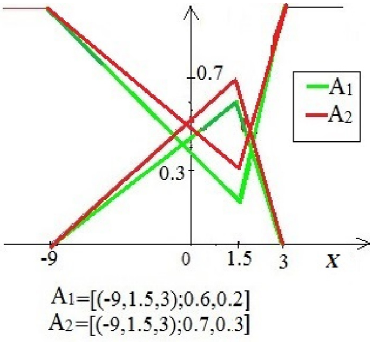

Fig. 2 Example 3.2

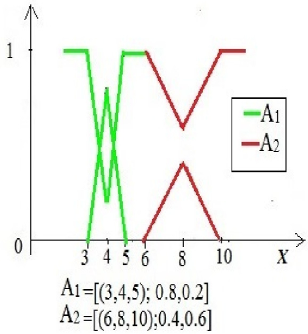

Fig. 3 Example 3.3

Here $v_\mu(A_i)$ and $v_\nu(A_i)$ are the values of the membership and non-membership functions of TIFN A_i and are defined as $v_\mu(A_i) = \frac{w_i(a_i+4b_i+c_i)}{6}$ and $v_\nu(A_i) = \frac{(1-u_i)(a_i+4b_i+c_i)}{6}$. On the other hand, $a_\mu(A_i)$ and $a_\nu(A_i)$ are the ambiguity indices of the membership and non-membership functions of TIFN A_i and are defined as: $a_\mu(A_i) = \frac{w_i(c_i-a_i)}{3}$ and $a_\nu(A_i) = \frac{(1-u_i)(c_i-a_i)}{3}$.

The parameter $\lambda \in [0, 1]$ represents the preference attitude of decision maker and significance of λ is given in [37]. Finally, the ranking can be done according to the non-increasing order of the ratio values $R(A_i)$. However, it is observed that this method still produces unreasonable ranking outcomes under certain situations. To show the limitations, following examples are provided.

Example 3.2 Consider TIFNs $A_1 = [(-9, 1.5, 3); 0.6, 0.2]$ and $A_2 = [(-9, 1.5, 3); 0.7, 0.3]$ as shown in Fig.2. By Li's method, the ratio of the value index and the ambiguity index of TIFNs A_1 and A_2 are $R(A_1) = R(A_2) = 0$. Thus, the ranking order of TIFNs A_1 and A_2 is the same, i.e., $A_1 \approx A_2$ for every $\lambda \in [0, 1]$. Therefore, Li's approach does not provide a satisfactory result.

In addition, it is also found that if TIFNs $A_1 = [(a_1, b_1, c_1); w_1, u_1]$ and $A_2 = [(a_2, b_2, c_2); w_2, u_2]$ satisfy $a_i + 4b_i + c_i = 0$ (for i=1, 2), then the ratio of value and ambiguity of A_1 and A_2 will be equal by Li's method, i.e., $R(A_1) = R(A_2) = 0$, that is, they can not be compared by using Li's approach.

Example 3.3 Consider TIFNs $A_1 = [(3, 4, 5); 0.8, 0.2]$ and $A_2 = [(6, 8, 10); 0.4, 0.6]$ as shown in Fig.3. The ranking result by Li's method is $R(A_1) = R(A_2) = 2.087$. This implies that TIFNs A_1 and A_2 are not comparable although they are not identical. Hence, the result obtained by Li's approach is not satisfactory.

3.2.2 A Review on Dubey and Mehra's Process

In order to compare n TIFNs A_i $(i = 1, 2, \cdots, n)$, Dubey and Mehra [38] proposed a ranking function which is defined as: $R(A_i) = v(A_i) - a(A_i)$, where the value index $v(A_i)$ and ambiguity index $a(A_i)$ are defined in a way similar to Li's process [37]. Finally, the ranking can be done according to the non-increasing order of $R(A_i)$. To show the drawback of this method, one example is given below.

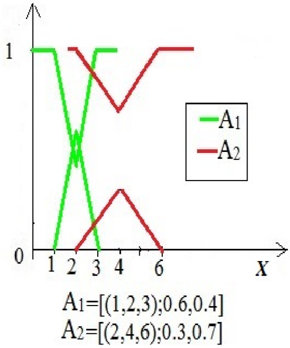

Fig. 4 Example 3.4

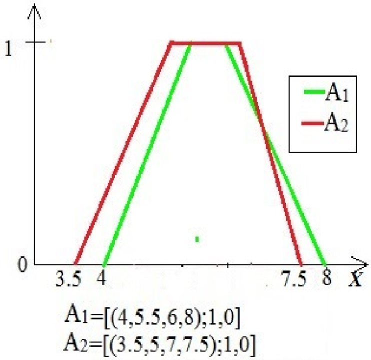

Fig. 5 Example 3.5

Example 3.4 Consider TIFNs $A_1 = [(1,2,3);0.6,0.4]$ and $A_2 = [(2,4,6);0.3,0.7]$ as shown in Fig.4. The ranking result according to this method is $R(A_1) = R(A_2) = 0.8$. Hence, these two TIFNs are not comparable although they are not identical. Therefore, Dubey and Mehra's approach does not provide a satisfactory result.

3.2.3. A Review on Rezvani's Process

In order to compare n TrIFNs $A_i = [(a_i, b_i, c_i, d_i); 1, 0]$ $(i = 1, 2, \cdots, n)$, a value index [39] for each TrIFN A_i is defined as: $v(A_i) = \frac{a_i+2b_i+2c_i+d_i}{6}$.

The ranking can be done according to the non-increasing order of their value indices $v(A_i)$. This method has also some limitations and following example is given to show this.

Example 3.5 Consider TrIFNs $A_1 = [(4,5.5,6,8);1,0]$ and $A_2 = [(3.5,5,7,7.5);1,0]$ as shown in Fig.5. The value indices of these two TrIFNs are $v(A_1) = 5.8333$ and $v(A_2) = 5.8333$. Hence, these two numbers are not comparable although they are not identical. Hence, the result obtained by Rezvani's process is not satisfactory.

3.3. Score and Accuracy Function Based Approach

In intuitionistic fuzzy framework, a series of score functions and accuracy functions were defined from various perspectives and based on the score and accuracy functions, ranking method of IFVs were developed in [5, 9, 10, 12, 54, 55]. The concept of score functions and accuracy functions were also widely applied to MCDM problems [5, 7, 56-58]. Score and accuracy functions are generally defined as the difference and the sum of the membership and non-membership functions, respectively.

3.3.1. A Review on Wang and Zhong's Process

For two TrIFNs $A_1 = [(a_1, b_1, c_1, d_1); w_1, u_1]$ and $A_2 = [(a_2, b_2, c_2, d_2); w_2, u_2]$, the ranking process based on score function ($s(A_i)$) and accuracy function ($h(A_i)$) [34] is as follows:

$$\text{if } s(A_1) > s(A_2), \text{ then } A_1 > A_2,$$

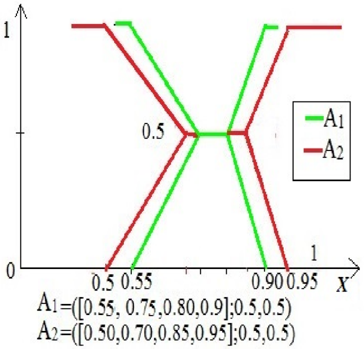

Fig. 6 Example 3.6

if $s(A_1) = s(A_2)$ and

if $h(A_1) = h(A_2)$, then $A_1 = A_2$,

if $h(A_1) > h(A_2)$, then $A_1 > A_2$,

where $s(A_i) = I(A_i) \times (w_i - u_i)$, $h(A_i) = I(A_i) \times (w_i + u_i)$ and $I(A_i) = \frac{1}{8}[(a_i + b_i + c_i + d_i) \times (1 + w_i - u_i)]$ are score functions, accuracy functions and expected values of A_i $(i = 1, 2)$, respectively.

However, after analyzing this method it is observed that, it has also limitation and this is shown by considering an example.

Example 3.6 Consider two TrIFNs, $A_1 = [(0.55, 0.74, 0.8, 0.9); 0.5, 0.5]$ and $A_2 = [(0.5, 0.7, 0.85, 0.95); 0.5, 0.5]$. By utilizing this method, the ranking result is $s(A_1) = 0 = s(A_2)$ and $h(A_1) = 0.375 = h(A_2)$. So, these two TrIFNS (Fig.6) are indifferent according to this process although they are not identical. Therefore, Wang and Zhong's process does not provide a satisfactory result.

The above observation provided in Example 3.1 to Example 3.6 inspired us to develop a new approach to compare and rank TrIFNs so that it can overcome the drawbacks of the existing methods.

4. The Proposed Ranking Method for TrIFNs

This section presents a new approach to compare TrIFNs, which adequately incorporates a method to compute centroid point of TrIFNs.

4.1. Centroid Point of TrIFNs

From last two decades, researchers have given attention to investigate the centroid point of fuzzy numbers and also used it for comparing and ranking fuzzy numbers. Yager [59] introduced the concept of centroid-based ranking method for comparing fuzzy numbers where he used only X coordinate of centroid point. Murakami et al. [60] proposed a centroid-based ranking approach that calculates the centroid point of fuzzy numbers and ranked them according to the larger value of X coordinate and (or) Y coordinate. Cheng [61] proposed a ranking method by using distance between original and centroid point. Chu and Tsao [62] proposed a ranking method of fuzzy numbers with an area between the centroid point and the original point. However, Wang et al. [63] demonstrated that the centroid formulae defined in [61] and [62] are incorrect and in their paper they provided the correct centroid formulae. Recently, Dat et al. [64] proposed an improved ranking method based on the centroid point for ranking various types of fuzzy numbers. In addition, centroid point has also been analyzed for other generalization of fuzzy sets [65, 66].

In the present study, we develop a ranking technique to rank TrIFNs by utilizing the concept of centroid point of TrIFNs. Nevertheless, computation of the centroid point of TrIFNs has not yet been attended so far. Therefore, inspired by the method of [63], we present the below method to compute the centroid point of TrIFNs by using the following steps.

Step 1: Computation of X coordinate of the centroid point.
Let $A = [(a,b,c,d); w,u]$ be a TrIFN, which is shown in Fig.7. Let $f_A^L : [a,b] \to [0,w]$ and $f_A^R : [c,d] \to [0,w]$ are the left and right parts of the membership function and $g_A^L : [a,b] \to [0,u]$ and $g_A^R : [c,d] \to [0,u]$ are left and right parts of non-membership function of TrIFN A, which are defined in Equations (1) and (2), respectively. Functions $f_A^L(x), f_A^R(x), g_A^L(x)$ and $g_A^R(x)$ can be analytically expressed by utilizing Equations (1) and (2) as follows:

$$f_A^L(x) = \frac{w(x-a)}{b-a} \text{ for } a \le x \le b; \; g_A^L(x) = \frac{(x-b)+u(a-x)}{(a-b)} \text{ for } a \le x \le b;$$

$$f_A^R(x) = \frac{w(d-x)}{(d-c)} \text{ for } c \le x \le d; \; g_A^R(x) = \frac{(x-c)+u(d-x)}{(d-c)} \text{ for } c \le x \le d.$$

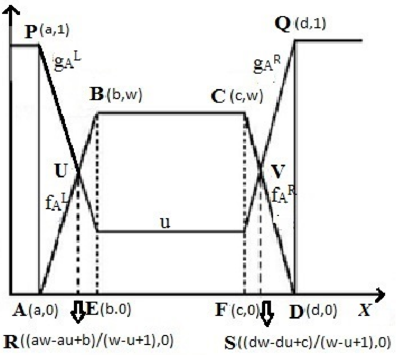

Fig. 7 TrIFN A

In order to determine the centroid point (X_A, Y_A) of TrIFN A, the area under the membership and non-membership functions is considered together [40]. To determine the X coordinate, first of all, the whole TrIFN is split into five rectangles (Fig.7): $ARUP$, $REBU$, $EFCB$, $FSVC$ and $SDQV$ where the coordinates of the corner points of each of the rectangles are computed as follows:

$$A:(a,0),\ B:(b,w),\ C:(c,w),\ D:(d,0),\ E:(b,0),$$

$$F:(c,0),\ Q:(d,1),\ R:\Big(\frac{aw-au+b}{w-u+1},0\Big),\ S:\Big(\frac{dw-du+c}{w-u+1},0\Big),$$

$$P:(a,1),\ U:\Big(\frac{aw-au+b}{w-u+1},\frac{w}{w-u+1}\Big),\ V:\Big(\frac{dw-du+c}{w-u+1},\frac{w}{w-u+1}\Big).$$

Then the X coordinate (X_A) of centroid point of TrIFN A can be computed by using the following formula:

$$X_A=\frac{\int_a^{R_x} xg_A^L dx+\int_{R_x}^{b} xf_A^L dx+\int_b^c xw dx+\int_c^{S_x} xf_A^R dx+\int_{S_x}^{d} xg_A^R dx}{\int_a^{R_x} g_A^L dx+\int_{R_x}^{b} f_A^L dx+\int_b^c w dx+\int_c^{S_x} f_A^R dx+\int_{S_x}^{d} g_A^R dx}, \tag{4}$$

where R_x and S_x are X coordinates of the points R and S respectively.

Step 2: Computation of Y coordinate of the centroid point.
To determine the Y coordinate of TrIFN A, the inverse function of TrIFN is taken and the whole area is split into three geometric areas (Fig.8): $ABCD$, DVQ and APU, where the coordinates of the corner points of geometric areas are provided in Step 1. As f_A^L, f_A^R, g_A^L and g_A^R are strictly monotonic and continuous functions, their inverse functions should exist and also be continuous and strictly monotonic. Let the inverse functions of f_A^L and f_A^R be $h_A^L:[0,w]\to[a,b]$ and $h_A^R:[0,w]\to[c,d]$ respectively, and $k_A^L:[0,u]\to[a,b]$ and $k_A^R:[0,u]\to[c,d]$ be the inverse functions of g_A^L and g_A^R, respectively. The inverse functions $h_A^L(y), h_A^R(y), k_A^L(y)$ and $k_A^R(y)$ can be analytically expressed (by using Equations (1) and (2)) as follows:

$$h_A^L(y)=a+\frac{(b-a)y}{w}\ \text{for } 0\le y\le w;\ k_A^L(y)=\frac{(a-b)y+(b-au)}{1-u}\ \text{for } u\le y\le 1;$$

$$h_A^R(y)=d-\frac{(d-c)y}{w}\ \text{for } 0\le y\le w;\ k_A^R(y)=\frac{(d-c)y+(c-du)}{1-u}\ \text{for } u\le y\le 1.$$

Then the Y coordinate (Y_A) of centroid point of TrIFN A can be computed by using

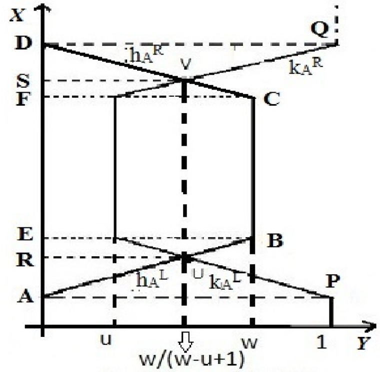

Fig. 8 Inverse of TrIFN A

the following formula

$$Y_A = \frac{\int\limits_0^w y\left(h_A^R - h_A^L\right)dy + \left[\int\limits_0^1 yd.dy - \int\limits_0^{V_y} yh_A^R dy - \int\limits_{V_y}^1 yk_A^R dy\right]}{\int\limits_0^w \left(h_A^R - h_A^L\right)dy + \left[\int\limits_0^1 d.dy - \int\limits_0^{V_y} h_A^R dy - \int\limits_{V_y}^1 k_A^R dy\right]} \quad \frac{+\left[\int\limits_0^{U_y} yh_A^L dy + \int\limits_{U_y}^1 yk_A^L dy - \int\limits_0^1 aydy\right]}{+\left[\int\limits_0^{U_y} h_A^L dy + \int\limits_{U_y}^1 k_A^L dy - \int\limits_0^1 ady\right]}, \tag{5}$$

where U_y and V_y are Y coordinates of the points U and V respectively.

In particular, if $w = 1$ and $u = 0$, then the formulae of centroid point become

$$X_A = \frac{(3a^2 + b^2 - c^2 - 3d^2)}{2(3a + b - c - 3d)}, \tag{6}$$

$$Y_A = \frac{7(d - a) + 5(c - b)}{18(d - a) + 6(c - b)}. \tag{7}$$

4.2. Rationality Validation of Proposed Centroid Formulae

In the following, an attempt is made to justify the proposed centroid Formulas (4) and (5).

Theorem 4.1 *Let A be a TrIFN with its centroid point (X_A, Y_A) which is computed by Equations* (4) and (5). *Suppose TrIFN B is right or left translation (α) of TrIFN*

A along X-axis. Then the centroid point of B moves to exactly the same distance (α) *along the X-axis, but the coordinate of its centroid on Y-axis remains unchanged, i.e.,* (a) $X_B = X_A + \alpha$ *and* (b) $Y_B = Y_A$.

Proof Proof of the part (a) is trivial. So, in order to prove part (b) we may proceed as follows:

Consider a TrIFN A with the membership function $\mu_A(x)$ and non-membership function $\nu_A(x)$ which can be written in the following form:

$$\mu_A(x) = \begin{cases} f_A^L(x), & \text{for } a \le x \le b, \\ w, & \text{for } b \le x \le c, \\ f_A^R(x), & \text{for } c \le x \le d, \\ 0, & \text{for } x \le a, x \ge d, \end{cases} \tag{8}$$

and

$$\nu_A(x) = \begin{cases} g_A^L(x), & \text{for } a \le x \le b, \\ u, & \text{for } b \le x \le c, \\ g_A^R(x), & \text{for } c \le x \le d, \\ 1, & \text{for } x \le a, x \ge d. \end{cases} \tag{9}$$

The membership function $\mu_B(x)$ and non-membership function $\nu_B(x)$ of TrIFN B can be defined as:

$$\mu_B(x) = \begin{cases} f_A^L(x \pm \alpha), & \text{for } a + \alpha \le x \le b + \alpha, \\ w, & \text{for } b + \alpha \le x \le c + \alpha, \\ f_A^R(x \pm \alpha), & \text{for } c + \alpha \le x \le d + \alpha, \\ 0, & \text{for } x \le a + \alpha, x \ge d + \alpha, \end{cases} \tag{10}$$

and

$$\nu_B(x) = \begin{cases} g_A^L(x \pm \alpha), & \text{for } a + \alpha \le x \ge b + \alpha, \\ u, & \text{for } b + \alpha \le x \le c + \alpha, \\ g_A^R(x \pm \alpha), & \text{for } c + \alpha \le x \le d + \alpha, \\ 1, & \text{for } x \le a + \alpha, x \ge d + \alpha. \end{cases} \tag{11}$$

where $\alpha(> 0)$ is a constant and $f_A^L(x), f_A^R(x), g_A^L(x)$ and $g_A^R(x)$ are defined in Step 1 of Section 4.1. The inverse functions of $f_A^L(x), f_A^R(x), g_A^L(x)$ and $g_A^R(x)$ are $h_A^L(y), h_A^R(y), k_A^L$ (y) and $k_A^R(y)$, respectively and they are defined in Step 2 of Section 4.1. Let $h_B^L(y), h_B^R(y)$, $k_B^L(y)$ and $k_B^R(y)$ be the inverse functions of $f_B^L(x), f_B^R(x), g_B^L(x)$ and $g_B^R(x)$, respectively.

It is known that translation does not change the shape of TrIFN B, so, for the right translation, the expressions of $h_B^L(y), h_B^R(y), k_B^L(y)$ and $k_B^R(y)$ can be written as

$$h_B^L(y) = h_A^L(y) + \alpha; \; h_B^R(y) = h_A^R(y) + \alpha,$$

$$k_B^L(y) = k_A^L(y) + \alpha; \; k_B^R(y) = k_A^R(y) + \alpha.$$

Now by using the centroid Formula (5), we may write

$$Y_B = \frac{\int_0^w y\left(h_A^R - \alpha - h_A^L + \alpha\right)dy + \left[\int_0^1 yd.dy - \int_0^{V_y} y(h_A^R - \alpha)dy - \int_{V_y}^1 y(k_A^R - \alpha)dy\right]}{\int_0^w \left(h_A^R - \alpha - h_A^L + \alpha\right)dy + \left[\int_0^1 d.dy - \int_0^{V_y} (h_A^R - \alpha)dy - \int_{V_y}^1 (k_A^R - \alpha)dy\right]}$$

$$\frac{+\left[\int_0^{U_y} y(h_A^L - \alpha)dy + \int_{U_y}^1 y(k_A^L - \alpha)dy - \int_0^1 aydy\right]}{+\left[\int_0^{U_y} (h_A^L - \alpha)dy + \int_{U_y}^1 (k_A^L - \alpha)dy - \int_0^1 ady\right]}$$

$$= \frac{\int_0^w y\left(h_A^R - h_A^L\right)dy + \left[\int_0^1 yd.dy - \int_0^{V_y} yh_A^R dy - \int_{V_y}^1 yk_A^R dy\right]}{\int_0^w \left(h_A^R - h_A^L\right)dy + \left[\int_0^1 d.dy - \int_0^{V_y} h_A^R dy - \int_{V_y}^1 k_A^R dy\right]}$$

$$\frac{+\left[\int_0^{U_y} yh_A^L dy + \int_{U_y}^1 yk_A^L dy - \int_0^1 aydy\right]}{+\left[\int_0^{U_y} h_A^L dy + \int_{U_y}^1 k_A^L dy - \int_0^1 ady\right]}$$

$$= Y_A.$$

Similarly, we can prove the results for the left translation.

Theorem 4.2 *Let A be a TrIFN and its centroid point is (X_A, Y_A) which is computed by Equations* (4) and (5). *Suppose TrIFN C is the magnification* (W) *of TrIFN A along the Y-axis. Then the variations of membership and non-membership along the Y-axis should make changes of Y_A and Y_C but not any changes of X_A and X_C, i.e., the centroid point of C moves along the Y-axis, but the coordinate of its centroid on the X-axis remains unchanged.*

Proof Consider a TrIFN C with its membership and non-membership functions $f_C(x)$ and $g_C(x)$ which are defined as $f_C(x) = Wf_A(x)$ and $g_C(x) = Wg_A(x)$, where $0 < W < 1$. So, TrIFN C must have the same centroid point on X-axis as TrIFN A. Now by using the centroid Formula (4), we have

$$X_C = \frac{\int_a^{R_x} xg_C^L dx + \int_{R_x}^b xf_C^L dx + \int_b^c xwdx + \int_c^{S_x} xf_C^R dx + \int_{S_x}^d xg_C^R dx}{\int_a^{R_x} g_C^L dx + \int_{R_x}^b f_C^L dx + \int_b^c wdx + \int_c^{S_x} f_C^R dx + \int_{S_x}^d g_C^R dx}$$

$$= \frac{W\left[\int_a^{R_x} xg_A^L dx + \int_{R_x}^b xf_A^L dx + \int_b^c xwdx + \int_c^{S_x} xf_A^R dx + \int_{S_x}^d xg_A^R dx\right]}{W\left[\int_a^{R_x} g_A^L dx + \int_{R_x}^b f_A^L dx + \int_b^c wdx + \int_c^{S_x} f_A^R dx + \int_{S_x}^d g_A^R dx\right]}$$

$$= \frac{\int_a^{R_x} x g_A^L dx + \int_{R_x}^{b} x f_A^L dx + \int_b^{c} xw dx + \int_c^{S_x} x f_A^R dx + \int_{S_x}^{d} x g_A^R dx}{\int_a^{R_x} g_A^L dx + \int_{R_x}^{b} f_A^L dx + \int_b^{c} w dx + \int_c^{S_x} f_A^R dx + \int_{S_x}^{d} g_A^R dx}$$

$$= X_A.$$

Hence the natural properties of centroid point are satisfied by Formulas (4) and (5).

4.3. A Centroid Point Based Ranking Algorithm

Let us consider a set Ω consisting of n TrIFNs $A_i, i = 1, 2, \cdots, n$. Inspired by the principle of compromise ratio index [44], an algorithm is proposed to compare and rank TrIFNs. The rationale of the proposed algorithm is that, a TrIFN is preferred if it's centroid point, computed through the proposed method, is closest to the PIS and the farthest away from NIS. The ranking process can be done according to the following steps.

Step 1: Computation of centroid point of each TrIFN.
Compute the centroid point of A_is by utilizing Equations (4) and (5) and let (X_{A_i}, Y_{A_i}) be the centroid point of $A_i, i = 1, 2, \cdots, n$. Let $\Lambda = \{C_1, C_2, \cdots, C_n\}$ be the set of centroid points of A_is, where $C_i = (X_{A_i}, Y_{A_i})$ is the corresponding centroid point of A_i.

Step 2: Determination of PIS and NIS.
Let PIS and NIS be defined as: $C^+ = (X_{\max}, Y_{\max}) = (\max_i\{X_{A_i}\}, \max_i\{Y_{A_i}\})$ and $C^- = (X_{\min}, Y_{\min}) = (\min_i\{X_{A_i}\}, \min_i\{Y_{A_i}\})$.

The definition of ideal solutions as above allow the consideration of the representative location (i.e., x coordinate of A_i) of the IFN and average height (i.e., y coordinate of A_i) of the IFN.

Step 3: Evaluation of the distances of TrIFNs from PIS and NIS by using Minkowski L_p metric [67].
The distance between C_i and PIS C^+ is measured as follows

$$d_p(C_i, C^+) = \sqrt[p]{|X_{\max} - X_{A_i}|^p + |Y_{\max} - Y_{A_i}|^p}, (i = 1, 2, \cdots, n).$$

where $p \geq 1$ is a distance parameter.

A centroid point C_i^* satisfying $d_p(C_i^*, C^+) = \min_{1 \leq i \leq n} d_p(C_i, C^+)$ should be the best as it has the shortest distance from PIS C^+. Thus, we may say the corresponding TrIFN A_i^* is closest to the PIS. However, such a TrIFN may not always have the longest distance from the NIS.

Similarly, the distance between C_i and NIS C^- is measured as follows

$$d_p(C_i, C^-) = \sqrt[p]{|X_{\min} - X_{A_i}|^p + |Y_{\min} - Y_{A_i}|^p}, (i = 1, 2, \cdots, n).$$

A centroid point C_i^{**} satisfying $d_p(C_i^{**}, C^-) = \max_{1 \leq i \leq n} d_p(C_i, C^-)$ is the centroid point of the best alternative which has the farthest distance from the NIS C^-.

However, a centroid point of a TrIFN which has the farthest distance from NIS may not always be guaranteed that it is closest to the PIS, i.e., it may not be happened that $C_i^* = C_i^{**}$.

Step 4: Computation of ranking index.
In this study, by attaching relative importance of both the distances $d_p(C_i, C^+)$ and $d_p(C_i, C^-)$, a compromise ratio ranking index is defined as follows

$$\begin{aligned} R(A_i) = {} & \omega \frac{\max\limits_{1\le i\le n} d_p(C_i, C^+) - d_p(C_i, C^+)}{\max\limits_{1\le i\le n} d_p(C_i, C^+) - \min\limits_{1\le i\le n} d_p(C_i, C^+)} \\ & +(1-\omega)\frac{d_p(C_i, C^-) - \min\limits_{1\le i\le n} d_p(C_i, C^-)}{\max\limits_{1\le i\le n} d_p(C_i, C^-) - \min\limits_{1\le i\le n} d_p(C_i, C^-)}, \end{aligned} \tag{12}$$

where the parameter ω is referred to as the decision maker's preference attitude and $\omega \in [0, 1]$.

Step 5: Ranking of TrIFNs.
Finally, a TrIFN A_j^0, whose centroid point satisfy the relation $R(A_j^0) = \max\limits_{1\le i\le n} R(A_i)$, should be the best 'compromise' solution which has the best compromise level between the distance from the PIS C^+ and the distance from the NIS C^-.

Step 6: End.

There are some important observations of the proposed ranking method which are summarized below in the form of remarks.

Remark 4.1 In the proposed ranking approach, the PIS C^+ may not be a feasible centroid point, i.e., $C^+ \notin \Lambda$. Otherwise, the corresponding TrIFN A^+ is the best one. Thus, in view of this, in the aforementioned algorithm, we assume that $C^+ \notin \Lambda$. Similarly, we assume that $C^- \notin \Lambda$.

Remark 4.2 In the proposed ranking index $R(A_i)$, the parameter ω indicates the degree of optimism/pessimism of the expert. In particular, from a pessimistic expert's point of view ω takes the value 1. The parameter ω takes the value 0, if the expert is optimistic. If $\omega = 0.5$, then the expert is unbiased. But in reality, it is quite reasonable to assume that ω takes any value in [0, 1].

Remark 4.3 In literature, TOPSIS based ranking index was utilized to compare and rank IFVs [3, 13]. In TOPSIS, the basic principle is that the chosen alternative should have the shortest distance from the PIS and the farthest distance from the NIS. However, later it is realized that the relative importance of the distances from PIS and NIS are not considered in the TOPSIS based ranking index, although this could be a major concern in decision making. Owing to this fact, inspired by compromise ratio methodology [44], a new aggregation function for ranking TrIFNs is introduced here to reflect the relative importance of distances from both PIS and NIS. In fact, this relative importance represents the compromise satisfactory level.

Remark 4.4 Wang and Kerre [68] proposed a set of reasonable ordering axioms $A_1 - A_7$ for validation of a ranking method for the ranking of fuzzy objects. It can easily be seen that, the proposed ranking process satisfies all the axioms $A_1 - A_5$. Since the ranking index (12) is not a linear function, axioms A_6 and A_7 are not satisfied by (12).

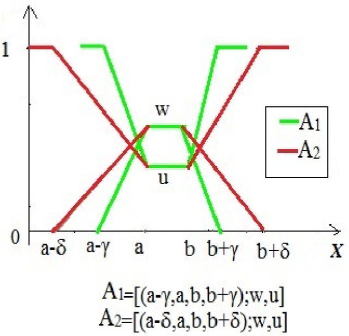

Fig. 9 Symmetric TrIFNs

Remark 4.5 It is worth noticing that the existing ranking methods of TrIFNs is to construct a mathematical function from the notion of TrIFNs, followed by the comparison among these mathematical functions to rank TrIFNs. Due to lack of strict theoretical analysis, all these methods sometimes produce inconsistent ranking results. The proposed ranking method in this paper can overcome these drawbacks. The reasons are shown as following: first, our ranking method use centroid points, it is simple and easy to use; second, two parameters, PIS and NIS, are used to compute the ranking index for TrIFNs, as these aspects of two parameters are equivalently important to produce a reasonable ranking result; third, the proposed ranking method satisfies reasonable ranking axioms [68], which means our method has a solid theoretical foundation; fourth, the proposed ranking method is straight forward and simple to use.

Remark 4.6 Let us consider two TrIFNs A_1 and A_2 of the following form $A_1 = [(a - \gamma, a, b, b + \gamma); w, u]$ and $A_2 = [(a - \delta, a, b, b + \delta); w, u]$ where $\gamma \neq \delta$. These two numbers have the same centroid point due to their symmetrical nature (Fig.9). In such case, ranking may be done according to their lower support values (defined in Definition 2.5), i.e., if support$(A_1) >$ support(A_2), then $A_1 < A_2$.

4.4. Results and Discussions

To present the rationality and necessity for proposing new ranking approach, Example 3.1 to Example 3.6 (Fig.1 to Fig.6) are considered (Table 2) to compare the proposed method with the other methods. The comparison results are presented in Table 2. From Table 2 and from Fig.1 to Fig.6 we can see some limitations of the existing methods and some advantages of the proposed method, which are illustrated below.

In the given examples, p is specified by $p = 2$ and without any loss of generality, ω is taken as 0.5.

- In Example 3.1 (Fig.1), it is observed that, TrIFNs A_1 and A_2 are not comparable by Wei's [35] process. This is an incorrect ranking result as A_1 and A_2 are not identical. The ranking result by utilizing the proposed method is $A_1 \succ A_2$. This result agrees with human intuition and the ranking results of [34, 37, 38]. However, the ranking result given by Rezvani [39] provides an inconsistent result.

- From Example 3.2 (Fig.2), it is clear that, by Rezvani's [39] and Li's [37] process, two TrIFNs A_1 and A_2 are not comparable whereas the ranking result obtained by the proposed method is consistent with the other approaches and the ranking result is $A_2 \succ A_1$.

- In Example 3.3 (Fig.3) and Example 3.4 (Fig.4), the ranking result by using the proposed method is $A_2 \succ A_1$ which is the same as Rezvani's [39] process. But by Wei's [35], Li's [37], Dubey and Mehra's [38] approaches, TrIFNs A_1 and A_2 are not comparable.

- For Example 3.5 (Fig.5), it is observed that, ranking result by using Rezvani's [39] process is $A_1 \approx A_2$ and the ranking result by using Wang and Zhong's [34] method is $A_1 \succ A_2$ which is not consistent with human intuitions. However, the proposed method, Wei's [35], Li's [37] and Dubey and Mehra's [38] methods provide a consistent result.

- From Example 3.6 (Fig.6), according to Wei's [35], Wang and Zhong's [34] and Rezvani's [39] processes ranking result is $A_1 \approx A_2$, i.e., TrIFNs A_1 and A_2 are not comparable, which is an incorrect ranking result as these two TrIFNs are not identical. The ranking result by utilizing the proposed method is $A_1 \succ A_2$. This result is reasonable and consistent with human intuition and the ranking results obtained by [37, 38].

These examples discussed above demonstrate that the proposed ranking method is able to overcome the demerits of the existing ranking processes and give more reasonable results for the comparisons of TrIFNs.

5. An Application of Proposed Ranking Process to Solve an MCDM Problem

In this section, an MCDM problem, where the decision information is quantified by TrIFNs, is presented based on the proposed ranking process. A multi-criteria decision process is designed with n criteria $C = \{C_1, C_2, \cdots, C_n\}$ to evaluate m number of alternatives $A = \{A_1, A_2, \cdots, A_m\}$. In general, criteria set C can be divided into two subsets P and Q in which P is the subset of benefit criteria and Q is the subset of cost criteria and $P \cup Q = C, P \cap Q = \phi$. The expert's intention is to select the alternative satisfying the higher value for benefit criteria and lower value for cost criteria, i.e., the expert's intention is to give the maximum rating to the alternative with maximum benefit and minimum cost. On the other hand, an alternative with maximum benefit

Table 2: Comparison result of the proposed ranking process with the existing methods.

Examples	Wei's process [35]	Wang and Zhong's process [34]	Rezvani's process [39]	Li's process [37]	Dubey and Mehra's process [38]	Proposed method
Ex-3.1: (See Fig.1) $A_1 = [(0.57, 0.73, 0.83); 0.73, 0.2]$ $A_2 = [(0.58, 0.74, 0.819); 0.72, 0.2]$	$\mathbf{A_1 \approx A_2}$	$A_1 \succ A_2$	$\mathbf{A_2 \succ A_1}$	$A_1 \succ A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$
Ex-3.2: (See Fig.2) $A_1 = [(-9, 1.5, 3); 0.6, 0.2]$ $A_2 = [(-9, 1.5, 3); 0.7, 0.3]$	$A_2 \succ A_1$	$A_2 \succ A_1$	$\mathbf{A_1 \approx A_2}$	$\mathbf{A_1 \approx A_2}$	$A_2 \succ A_1$	$A_2 \succ A_1$
Ex-3.3: (See Fig.3) $A_1 = [(3, 4, 5); 0.8, 0.2]$ $A_2 = [(6, 8, 10); 0.4, 0.6]$	$\mathbf{A_1 \approx A_2}$	$\mathbf{A_1 \succ A_2}$	$A_2 \succ A_1$	$\mathbf{A_1 \approx A_2}$	$\mathbf{A_1 \approx A_2}$	$A_2 \succ A_1$
Ex-3.4: (See Fig.4) $A_1 = [(1, 2, 3); 0.6, 0.4]$ $A_2 = [(2, 4, 6); 0.3, 0.7]$	$\mathbf{A_1 \approx A_2}$	$\mathbf{A_1 \succ A_2}$	$A_2 \succ A_1$	$\mathbf{A_1 \approx A_2}$	$\mathbf{A_1 \approx A_2}$	$A_2 \succ A_1$
Ex-3.5: (See Fig.5) $A_1 = [(4, 5.5, 6, 8); 1, 0]$ $B = [(3.5, 5, 7, 7.5); 1, 0]$	$A_1 \succ A_2$	$\mathbf{A_2 \succ A_1}$	$\mathbf{A_1 \approx A_2}$	$A_1 \succ A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$
Ex-3.6: (See Fig.6) $A_1 = [(0.55, 0.75, 0.8, 0.9); 0.5, 0.5]$ $A_2 = [(0.5, 0.7, 0.85, 0.95); 0.5, 0.5]$	$\mathbf{A_1 \approx A_2}$	$\mathbf{A_1 \approx A_2}$	$\mathbf{A_1 \approx A_2}$	$A_1 \succ A_2$	$A_1 \succ A_2$	$A_1 \succ A_2$

Note: The results that are not satisfactory are given in bold.

(or minimum cost) should have maximum rating. Let $t = (t_1, t_2, \cdots, t_n)^T$ be the weight vector of criteria where $t_j \in [0, 1]$, $j = 1, 2, \cdots, n$ and $\sum_{j=1}^{n} t_j = 1$.

Suppose TrIFN $A_{ij} = [(a_{ij}, b_{ij}, c_{ij}, d_{ij}); w_{ij}, u_{ij}]$ denotes the linguistic expression of the expert corresponding to j^{th} criteria C_j $(j = 1, 2, \cdots, n)$ to evaluate i^{th} alternative A_i $(i = 1, 2, \cdots, m)$. Here w_{ij} and u_{ij} represent the corresponding degree of satisfaction and degree of dissatisfaction of the expert's opinion. So, MCDM problem with TrIFNs can be given in a matrix form as follows:

$$\mathcal{D} = (A_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{pmatrix} \end{array}.$$

The steps of the MCDM process can be described in the following way:

Step 1: Computation of the normalized decision matrix.
To avoid the effect of considering n criteria from different physical dimensions, the decision matrix $\mathcal{D}$ needs to be normalized. In order to measure all criteria in a comparable scale and to facilitate inter-criteria comparisons, the primary task is to normalize the decision matrix $\mathcal{D}$. The normalized decision matrix can be computed by using the following formulae

$$r_{ij} = \begin{cases} \Big[\Big(\dfrac{a_{ij}}{a_j^{\max}}, \dfrac{b_{ij}}{a_j^{\max}}, \dfrac{c_{ij}}{a_j^{\max}}, \dfrac{d_{ij}}{a_j^{\max}}\Big); w_{ij}, u_{ij}\Big], & \text{for } i = 1, \cdots, m,\ j \in P, \\ \\ \Big[\Big(1 - \dfrac{d_{ij}}{a_j^{\max}}, 1 - \dfrac{c_{ij}}{a_j^{\max}}, 1 - \dfrac{b_{ij}}{a_j^{\max}}, 1 - \dfrac{a_{ij}}{a_j^{\max}}\Big); w_{ij}, u_{ij}\Big], & \text{for } i = 1, \cdots, m,\ j \in Q, \end{cases} \tag{13}$$

where P and Q are the sets of benefit and cost criteria and $a_j^{\max} = \max\{a_{ij}, b_{ij}, c_{ij}, d_{ij}\}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$. After normalization, the decision matrix $\mathcal{D}$ is transformed into normalized decision matrix

$$\mathcal{N} = (r_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix} \end{array},$$

where r_{ij} can be written as: $r_{ij} = [(r_{ij}^1, r_{ij}^2, r_{ij}^3, r_{ij}^4); w_{ij}, u_{ij}]$.

Step 2: Formulation of the weighted normalized TrIFN decision matrix.
Considering degree of importance t_j of the criteria C_j, $\mathcal{N}$ is transformed into the weighted normalized decision matrix

$$\mathcal{W} = (R_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \dots & \vdots \\ R_{m1} & R_{m2} & \cdots & R_{mn} \end{pmatrix} \end{array}.$$

Here R_{ij} can be calculated by using Definition 2.4 as

$$R_{ij} = t_j \times r_{ij} = [(R^1_{ij}, R^2_{ij}, R^3_{ij}, R^4_{ij}); w_{ij}, u_{ij}], \tag{14}$$

where $R^k_{ij} = t_j r^k_{ij}, k = 1, \cdots, 4, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n.$

Step 3: Computation of final aggregated values for each alternative.
Weighted aggregated values with respect to each alternative A_i is determined by using TrIFWAM operator (Equation (3)) as

$$\mathcal{S}_i = \sum_{j=1}^{n} R_{ij} = \Big[\Big(\sum_{j=1}^{n} R^1_{ij}, \sum_{j=1}^{n} R^2_{ij}, \sum_{j=1}^{n} R^3_{ij}, \sum_{j=1}^{n} R^4_{ij}\Big); \min_j\{w_{ij}\}, \max_j\{u_{ij}\}\Big]. \tag{15}$$

Step 4: Ranking of the alternatives.
We compute the centroid point of final aggregated value S_i for each alternative A_i by using Equations (4) and (5). Finally, ranking of all the alternatives is done according to non-increasing order of the proposed ranking index defined in Equation (12).

Step 5: End.

6. A Numerical Example

In this section, a numerical example is described to illustrate the application of the proposed ranking method. To install an aerospace research organization center, one key factor is choosing a best location. Suppose government of India is looking for a location for the same. After preliminary survey, space scientists' group finds five locations A_1, A_2, A_3, A_4 and A_5. Space scientists' group asses the locations A_i ($i = 1, \cdots, 5$) on the basis of following five criteria: geographical position C_1, climate condition C_2, safety factor C_3, functional area C_4 and pollution factor C_5. Let $t = (0.2, 0.15, 0.25, 0.1, 0.3)^T$ be the weight vector of criteria C_j ($j = 1, \cdots, 5$). The rating of alternatives (locations) on the basis of criteria is given in Table 6.1 (Data are adapted from [34]).

Table 3: TrIFN decision matrix for five alternatives.

	Criteria				
Alt.	C_1	C_2	C_3	C_4	C_5
A_1	$[(1,2,3,4); 0.7,0.3]$	$[(5,6,7,8); 0.7,0.3]$	$[(3,4,5,6); 0.7,0.3]$	$[(4,5,7,8); 0.6,0.3]$	$[(4,5,6,7); 0.8,0.0]$
A_2	$[(2,3,4,5); 0.6,0.3]$	$[(6,7,8,9); 0.8,0.1]$	$[(4,5,6,7); 0.8,0.2]$	$[(3,4,5,6); 0.7,0.3]$	$[(6,7,8,9); 0.6,0.3]$
A_3	$[(1,2,3,5); 0.6,0.4]$	$[(4,6,7,8); 0.6,0.3]$	$[(3,4,5,6); 0.5,0.5]$	$[(4,5,6,7); 0.8,0.1]$	$[(5,6,7,8); 0.8,0.2]$
A_4	$[(2,3,4,6); 0.6,0.2]$	$[(5,6,7,8); 0.8,0.2]$	$[(2,3,5,6); 0.6,0.4]$	$[(3,4,5,7); 0.6,0.3]$	$[(4,6,7,8); 0.6,0.3]$
A_5	$[(2,3,4,5); 0.8,0.2]$	$[(4,5,6,7); 0.9,0.0]$	$[(3,4,5,6); 0.8,0.2]$	$[(3,5,7,8); 0.7,0.1]$	$[(4,5,6,7); 0.8,0.0]$

Step 1: The criteria are classified into two groups: criteria $C_1 - C_4$ are classified as benefit criteria. The cost criteria is C_5. So, by using the Equation (13), the normalized TrIFN decision matrix is obtained, which is shown in Table 4.

Step 2: By using the weight vector $t = (0.2, 0.15, 0.25, 0.1, 0.3)^T$ and Table 4, the weighted normalized decision matrix is computed and shown in Table 5.

Step 3: Again, by using Equation (15) and Table 5, the final aggregated values of the alternatives A_i ($i = 1,2,3,4,5$) are shown in Table 6.

Step 4: Finally, the decision results are obtained by using the proposed ranking method and the ranking order is presented in Table 6. Without loss of generality, the expert's attitude is specified by $w = 0.5$.

Hence, according to the above result, we have $A_5 > A_2 > A_1 > A_4 > A_3$, i.e., location A_5 will be the best choice, A_2 is the second choice, A_1 the third, A_4 the fourth and A_3 is the fifth choice.

Table 4: The normalized TrIFN decision matrix for five alternatives.

	Criteria				
Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	$[(0.167, 0.333, 0.5, 0.667); 0.7, 0.3]$	$[(0.556, 0.667, 0.778, 0.889); 0.7, 0.3]$	$[(0.429, 0.571, 0.714, 0.857); 0.7, 0.3]$	$[(0.5, 0.625, 0.875, 1.0); 0.6, 0.3]$	$[(0.222, 0.333, 0.444, 0.556); 0.8, 0.0]$
A_2	$[(0.333, 0.5, 0.667, 0.833); 0.6, 0.3]$	$[(0.667, 0.778, 0.889, 1.0); 0.8, 0.1]$	$[(0.571, 0.714, 0.857, 1.0); 0.8, 0.2]$	$[(0.375, 0.5, 0.625, 0.75); 0.7, 0.3]$	$[(0.000, 0.111, 0.222, 0.333); 0.6, 0.3]$
A_3	$[(0.167, 0.333, 0.5, 0.833); 0.6, 0.4]$	$[(0.444, 0.667, 0.778, 0.889); 0.6, 0.3]$	$[(0.429, 0.571, 0.714, 0.857); 0.5, 0.5]$	$[(0.50, 0.625, 0.75, 0.875); 0.8, 0.1]$	$[(0.111, 0.222, 0.333, 0.444); 0.8, 0.2]$
A_4	$[(0.333, 0.5, 0.667, 1.0); 0.6, 0.2]$	$[(0.556, 0.667, 0.778, 0.889); 0.8, 0.2]$	$[(0.286, 0.429, 0.714, 0.857); 0.6, 0.4]$	$[(0.375, 0.5, 0.625, 0.875); 0.6, 0.3]$	$[(0.111, 0.222, 0.333, 0.556); 0.6, 0.3]$
A_5	$[(0.333, 0.5, 0.667, 0.833); 0.8, 0.2]$	$[(0.444, .556, 0.667, 0.778); 0.9, 0.0]$	$[(0.429, 0.571, 0.714, 0.857); 0.8, 0.2]$	$[(0.375, .625, 0.875, 1.0); 0.7, 0.1]$	$[(0.222, 0.333, 0.444, 0.556); 0.8, 0.0]$

Table 5: The weighted normalized TrIFN decision matrix of five alternatives.

	Criteria				
Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	$[(0.0334, 0.0666, 0.1, 0.01334); 0.7, 0.3]$	$[(0.0834, .1001, .1167, 0.1334); 0.7, 0.3]$	$[(0.1073, .1428, .1785, 0.2143); 0.7, 0.3]$	$[(0.05, 0.0625, .0875, 0.1); 0.6, 0.3]$	$[(0.066, .0999, .1332, 0.1668); 0.8, 0.0]$
A_2	$[(0.0666, 0.1, 0.1334, 0.1666); 0.6, 0.3]$	$[(0.1001, .1167, .1334, 0.150); 0.8, 0.1]$	$[(0.1428, .1785, .2143, 0.25); 0.8, 0.2]$	$[(0.0375, 0.05, .0625, 0.075); 0.7, 0.3]$	$[(0.00, 0.333, 0.0666, 0.0999); 0.6, 0.3]$
A_3	$[(0.0334, 0.0666, 0.1, 0.1666); 0.6, 0.4]$	$[(0.0666, .1001, .1167, 0.1334); 0.6, 0.3]$	$[(0.1073, .1428, .1785, 0.2143); 0.5, 0.5]$	$[(0.05, 0.0625, .075, 0.0875); 0.8, 0.1]$	$[(0.033, .0666, .0999, 0.1332); 0.8, 0.2]$
A_4	$[(0.0666, 0.1, 0.1334, 0.200); 0.6, 0.2]$	$[(0.0834, .1001, .1167, 0.1334); 0.8, 0.2]$	$[(0.0715, .1073, .1785, 0.2143); 0.6, 0.4]$	$[(0.0375, 0.05, .0625, 0.0875); 0.6, 0.3]$	$[(0.033, .0666, .0999, 0.1668); 0.6, 0.3]$
A_5	$[(0.0666, 0.1, 0.1334, 0.1666); 0.8, 0.2]$	$[(0.0666, .0834, .1001, 0.1167); 0.9, 0.0]$	$[(0.1073, .1428, .1785, 0.2143); 0.8, 0.2]$	$[(0.037, .0625, .0875, 0.1); 0.7, 0.1]$	$[(0.066, .0999, .1332, 0.1668); 0.8, 0.0]$

Table 6: Final aggregation values of five alternatives and decision result.

Alter-natives	Final aggregation	Centroid point	Ranking value	Ranking order
A_1	$\mathcal{S}_1 = [(0.3407, 0.4718, 0.6159, 0.7478); 0.6, 0.3]$	$X_{\mathcal{S}_1} = 0.5443$, $Y_{\mathcal{S}_1} = 0.3414$	$R_{A_1} = 0.5512$	$A_5 > A_2 > A_1 > A_4 > A_3$
A_2	$\mathcal{S}_2 = [(0.3469, 0.4785, 0.6101, 0.7415); 0.6, 0.3]$	$X_{\mathcal{S}_2} = 0.5442$, $Y_{\mathcal{S}_2} = 0.3426$	$R_{A_2} = 0.5631$	
A_3	$\mathcal{S}_3 = [(0.2905, 0.4385, 0.5701, 0.7349); 0.5, 0.5]$	$X_{\mathcal{S}_3} = 0.5143$, $Y_{\mathcal{S}_3} = 0.3585$	$R_{A_3} = 0.0414$	
A_4	$\mathcal{S}_4 = [(0.2923, 0.4239, 0.5910, 0.7959); 0.6, 0.4]$	$X_{\mathcal{S}_4} = 0.5240$, $Y_{\mathcal{S}_4} = 0.3526$	$R_{A_4} = 0.1244$	
A_5	$\mathcal{S}_5 = [(0.3446, 0.4886, 0.6327, 0.7644); 0.7, 0.2]$	$X_{\mathcal{S}_5} = 0.5544$, $Y_{\mathcal{S}_5} = 0.3551$	$R_{A_5} = 1$	

6.1. Comparative Analysis

6.1.1. Comparison Analysis with the Existing Ranking Approaches

First, we compare the proposed ranking method with other existing approaches, such as, Wei's process [35], Li's process [37], Dubey and Mehra's process [38], Rezvani's process [39], Wang and Zhong's approach [34], by using the aerospace location selection problem described in Section 6. This MCDM example is solved by the aforementioned approaches and the results are shown in Table 7.

Table 7: The ranking order of different alternatives.

Ranking methods	Ranking results
Li's process [37]	$A_5 > A_1 > A_2 > A_4 > A_3$
Dubey and Mehra's process [38]	$A_5 > A_1 > A_2 > A_4 > A_3$
Wei's process [35]	$A_5 > A_1 > A_2 > A_4 > A_3$
Rezvani's process [39]	$A_5 > A_2 > A_1 > A_4 > A_3$
Wang and Zhong's approach [34]	$A_1 > A_5 > A_2 > A_4 > A_3$
Proposed ranking approach	$A_5 > A_1 > A_2 > A_4 > A_3$

As can be seen from Table 7, the ranking of the alternatives obtained by different methods are almost similar as it did by the proposed ranking approach which ensures that the proposed ranking approach works well.

In the following section, we compare proposed model with the corresponding fuzzy model and intuitionistic fuzzy value model.

6.1.2. Comparison Analysis with IFVs Based MCDM Model

As mentioned earlier, TrIFN is a further generalization of IFV and its universe is continuous rather than discrete, so its use is more convenient. In order to make the comparison, we use IFVs to express expert's opinion by assuming expert's satisfaction and dissatisfaction degrees (i.e., (w_{ij}, u_{ij})) of his(her) original opinion modeled by TrIFN A_i $(i = 1, 2, 3, 4, 5)$, as an intuitionistic fuzzy value ratings and thus, Table 3 can be written as Table 8.

Table 8: IFV decision matrix of five alternatives.

	Criteria				
Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	$(0.7, 0.3)$	$(0.7, 0.3)$	$(0.7, 0.3)$	$(0.6, 0.3)$	$(0.9, 0.0)$
A_2	$(0.6, 0.3)$	$(0.8, 0.1)$	$(0.8, 0.2)$	$(0.7, 0.3)$	$(0.6, 0.3)$
A_3	$(0.6, 0.4)$	$(0.6, 0.3)$	$(0.5, 0.5)$	$(0.8, 0.1)$	$(0.8, 0.2)$
A_4	$(0.6, 0.2)$	$(0.8, 0.2)$	$(0.6, 0.4)$	$(0.6, 0.3)$	$(0.6, 0.3)$
A_5	$(0.8, 0.2)$	$(0.7, 0.0)$	$(0.8, 0.2)$	$(0.7, 0.1)$	$(0.8, 0.0)$

By utilizing the weighted intuitionistic fuzzy arithmetic mean operator [69], we aggregate the intuitionistic fuzzy value ratings of the expert. The final aggregated values of alternatives A_i $(i = 1, 2, 3, 4, 5)$ are as follows:

$$\mathcal{S}_1 = (0.7787, 0.0), \mathcal{S}_2 = (0.7055, 0.2299), \mathcal{S}_3 = (0.6795, 0.2864),$$

$$\mathcal{S}_4 = (0.6395, 0.2797) \text{ and } \mathcal{S}_5 = (0.778, 0.0).$$

Finally, the decision results are obtained by using score function [9]. The ranking order of the alternatives is obtained as follows: $A_1 > A_5 > A_2 > A_3 > A_4$, i.e., location A_1 will be the best choice.

It is to be noted that the resultant ranking order quite differs from the ranking order obtained by our proposed method with TrIFNs information. This deviation mainly occurs due to the conversion of original TrIFNs information into intuitionistic fuzzy values. In other words, due to this transformation, TrIFN losses its inherent structure as all TrFNs are discarded from the corresponding TrIFNs. Such a conversion distorts expert's original assessments and also weakens the ability of information representation of TrIFNs. As TrIFNs provide a framework to maintain the integrity in information processing, therefore, TrIFNs may better capture expert's subjective estimation in a decision making problem than IFVs. Therefore, we urge that MCDM problems with TrIFNs data provide more appropriate results.

6.1.3. Comparison Analysis with TrFN Based MCDM Problem

In this section, the proposed method is compared with the corresponding TrFN based method. By adapting the membership function with full satisfaction in original ratings of the alternatives, i.e., by putting $w_{ij} = 1$ and $u_{ij} = 0$ ($\forall \ j = 1, 2, \cdots, n; i = 1, 2, \cdots, m$) TrIFN decision matrix (provided in Table 3) is transformed to TrFN decision matrix (provided in Table 9).

Table 9: TrFN decision matrix of five alternatives.

	Criteria				
Alternatives	C_1	C_2	C_3	C_4	C_5
A_1	$[1, 2, 3, 4]$	$[5, 6, 7, 8]$	$[3, 4, 5, 6]$	$[4, 5, 7, 8]$	$[4, 5, 6, 7]$
A_2	$[2, 3, 4, 5]$	$[6, 7, 8, 9]$	$[4, 5, 6, 7]$	$[3, 4, 5, 6]$	$[6, 7, 8, 9]$
A_3	$[1, 2, 3, 5]$	$[4, 6, 7, 8]$	$[3, 4, 5, 6]$	$[4, 5, 6, 7]$	$[5, 6, 7, 8]$
A_4	$[2, 3, 4, 6]$	$[5, 6, 7, 8]$	$[2, 3, 5, 6]$	$[3, 4, 5, 7]$	$[4, 6, 7, 8]$
A_5	$[2, 3, 4, 5]$	$[4, 5, 6, 7]$	$[3, 4, 5, 6]$	$[3, 5, 7, 8]$	$[4, 5, 6, 7]$

By utilizing the weighted trapezoidal fuzzy arithmetic mean operator [70], we aggregate the TrFN ratings of the expert. The final aggregated values of the alternatives A_i ($i = 1, 2, 3, 4, 5$) are as follows:

$$\mathcal{S}_1 = [0.3407, 0.4718, 0.6159, 0.7478], \mathcal{S}_2 = [0.3469, 0.4785, 0.6101, 0.7415],$$

$$\mathcal{S}_3 = [0.2905, 0.4385, 0.5701, 0.7349], \mathcal{S}_4 = [0.2923, 0.4239, 0.591, 0.7959],$$

$$\mathcal{S}_5 = [0.3446, 0.4886, 0.6327, 0.7644].$$

Finally, the decision results are obtained by using the proposed ranking method defined in Section 4 (using Equations (6), (7) and (12)). The ranking results of the alternatives are obtained as follows: $A_5 > A_4 > A_1 > A_2 > A_3$.

The ranking order by using TrFN based method quite differs from our proposed TrIFNs based method. This result shows that expert's satisfaction, dissatisfaction, hesitation related to human thinking, reasoning, etc. play an important role in decision results of MCDM problems. This uncertainty related to expert's subjective assessment are captured by TrIFNs in a logical and meaningful way and, therefore, MCDM with TrIFNs data provides more appropriate ranking results.

7. Conclusion

In this paper, we propose a new method for ranking of TrIFNs by utilizing the concept of centroid point of TrIFNs. We justify rationality of the proposed centroid formulas. Moreover, we provide examples to compare ranking results of the proposed method with the existing ranking processes of TrIFNs. The results indicate that, our proposed ranking method is able to provide reasonable ranking results. To establish the process of the proposed ranking method, an illustration for solving an MCDM problem has been given. Finally, with the help of the selection problem in aerospace research organization center, the feasibility and validity of the proposed ranking method have been illustrated. Applicability of modeling expert's opinion by TrIFNs has also been demonstrated in this article.

The advantages of this ranking method can be pointed out as follows: (i) the proposed ranking method is based on TrIFN and it is owing to the fact that TrIFN suitably reflects the uncertainty and hesitation of human thinking, thus, it gives more flexibility to the experts while expressing their opinions regarding each alternative over the criteria; (ii) the TrIFN representation of linguistic variables enable the experts to express their judgements with the corresponding degrees of satisfaction and dissatisfaction; (iii) the proposed ranking method adequately incorporates a method to compute the centroid point of TrIFNs; (iv) the proposed centroid point based ranking algorithm allows incorporation of the expert's attitudinal factor in the ranking index which makes the proposed scheme more flexible.

Although the proposed ranking approach is illustrated by a numerical selection problem of aerospace research organization center, it can also be applied to any other areas of decision problems where uncertainty and hesitation are involved in the evaluation process. This will be the topic of our future research work.

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References

[1] L.A. Zadeh, Fuzzy sets, Information and Control 8(3) (1965) 338-353.

[2] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1) (1986) 87-96.

[3] X.M. Zhang, Z.S. Xu, A new method for ranking intuitionistic fuzzy values and its application in multi-attribute decision making, Fuzzy Optimization and Decision Making 12 (2012) 135-146.

[4] F.E. Boran, S. Genç, M. Kurt, D. Akay, A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method, Expert Systems with Applications 36(8) (2009) 11363-11368.

[5] Z. Pei, L. Zheng, A novel approach to multi-attribute decision making based on intuitionistic fuzzy sets, Expert Systems with Applications 39(3) (2012) 2560-2566.

[6] T.Y. Chen, A comparative analysis of score functions for multiple criteria decision making in intuitionistic fuzzy settings, Information Sciences 181(17) (2011) 3652-3676.

[7] D.F. Li, Multiattribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets, Expert Systems with Applications 37(12) (2010) 8673-8678.

[8] S. Das, B. Dutta, D. Guha, Weight computation of criteria in a decision-making problem by knowledge measure with intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set, Soft Computing (2015) 1-22, doi: 10.1007/s00500-015-1813-3.

[9] S.M. Chen, J.M. Tan, Handling multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems 67(2) (1994) 163-172.

[10] F. Li, Y. Rao, Weighted methods of multi-criteria fuzzy decision making based on vague sets, Computer Science 28(7) (2001) 60-65.

[11] L.H. Chen, C.C. Tu, Dual bipolar measures of Atanassov's intuitionistic fuzzy sets, IEEE Transactions on Fuzzy Systems 22 (2014) 996-982.

[12] Z.S. Xu, R.R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, International Journal of General Systems 35(4) (2006) 417-433.

[13] Z.S. Xu, R.R. Yager, Dynamic intuitionistic fuzzy multi-attribute decision making, International Journal of Approximate Reasoning 48 (2008) 246-262.

[14] X. Yu, Z.S. Xu, S. Liu, Q. Chen, On ranking of intuitionistic fuzzy values based on dominance relations, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 22 (2014) 315-335.

[15] Z.S. Xu, Intuitionistic fuzzy multiattribute decision making: An interactive method, IEEE Transactions on Fuzzy Systems 20 (2012) 514-525.

[16] Z.S. Xu, R.R. Yager, Intuitionistic and interval-valued intutionistic fuzzy preference relations and their measures of similarity for the evaluation of agreement within a group, Fuzzy Optimization and Decision Making 8 (2009) 123-139.

[17] L. Huawen, S. Kaiquan, Intuitionistic fuzzy numbers and intuitionistic distribution numbers, Journal of Fuzzy Mathematics 8(4) (2000) 909-918.

[18] P. Grzegorzewski, Distances and orderings in a family of intuitionistic fuzzy numbers, EUSFLAT Conference, Citeseer, 2003, pp.223-227.

[19] P. Burillo, H. Bustince, V. Mohedano, Some definitions of intuitionistic fuzzy number. First properties, Proceedings of the 1st Workshop on Fuzzy Based Expert Systems, 1994, pp.53-55.

[20] R. Parvathi, C. Malathi, Arithmetic operations on symmetric trapezoidal intuitionistic fuzzy numbers, International Journal of Soft Computing and Engineering 2 (2012) 268-273.

[21] D. Guha, D. Chakraborty, A theoretical development of distance measure for intuitionistic fuzzy numbers, International Journal of Mathematics and Mathematical Sciences 2010 (2010) 1-25.

[22] J.Q. Wang, R. Nie, H.Y. Zhang, X.H. Chen, New operators on triangular intuitionistic fuzzy numbers and their applications in system fault analysis, Information Sciences 251 (2013) 79-95.

[23] J. Wu, Q.W. Cao, Same families of geometric aggregation operators with intuitionistic trapezoidal fuzzy numbers, Applied Mathematical Modelling 37 (2013) 318-327.

[24] S.P. Wan, Power average operators of trapezoidal intuitionistic fuzzy numbers and application to multi-attribute group decision making, Applied Mathematical Modelling 37 (2013) 4112-4126.

[25] J. Ye, Multicriteria decision-making method based on a cosine similarity measure between trapezoidal fuzzy numbers, International Journal of Engineering, Science and Technology 3 (2011) 272-278.

[26] J. Ye, Multicriteria group decision-making method using vector similarity measures for trapezoidal intuitionistic fuzzy numbers, Group Decision and Negotiation 21 (2012) 519-530.

[27] S. Das, D. Guha, Weight computation of alternatives in a decision-making problem by preference relation, Proceedings of Fourth International Conference on Soft Computing for Problem Solving, Springer, vol. 336, 2015, pp.25-36.

[28] S.P. Wan, J.Y. Dong, Power geometric operators of trapezoidal intuitionistic fuzzy numbers and application to multi-attribute group decision making, Applied Soft Computing 29 (2015) 153-168.

[29] S. Das, D. Guha, Similarity measure of intuitionistic fuzzy numbers by the centroid point, in: Mathematics and Computing 2013, vol. 91, Springer, 2014, pp.231-242.

[30] G.W. Wei, An approach to multiple attribute group decision making with interval intuitionistic trapezoidal fuzzy information, Technological and Economic Development of Economy 18 (2012) 317-330.

[31] G.W. Wei, X.F. Zhao, H.J. Wang, R. Lin, GRA model for selecting an ERP system in trapezoidal intuitionistic fuzzy setting, Information-An International Interdisciplinary Journal 13 (2010) 1143-1148.

[32] V. Nayagam, G. Venkateshwari, G. Sivaraman, Ranking of intuitionistic fuzzy numbers, IEEE International Conference on Fuzzy Systems, FUZZ-IEEE 2008, pp.1971-1974.

[33] V. Nayagam, G. Venkateshwari, G. Sivaraman, Modified ranking of intuitionistic fuzzy numbers, Notes on Intuitionistic Fuzzy Sets 17(1) (2011) 5-22.

[34] J.Q. Wang, Z. Zhong, Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems, Journal of Systems Engineering and Electronics 20(2) (2009) 321-326.

[35] G. Wei, Some arithmetic aggregation operators with intuitionistic trapezoidal fuzzy numbers and their application to group decision making, Journal of Computers 5(3) (2010) 345-351.

[36] C.P. Wei, X. Tang, Possibility degree method for ranking intuitionistic fuzzy numbers, International Conference on Web Intelligence and Intelligent Agent Technology (WI-IAT), IEEE/WIC/ACM, 3, 2010, pp.142-145.

[37] D.F. Li, A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problems, Computers and Mathematics with Applications 60(6) (2010) 1557-1570.

[38] D. Dubey, A. Mehra, Fuzzy linear programming with triangular intuitionistic fuzzy number. Advances in Intelligent System Research, EUSFLAT-LFA 2011, pp. 563-569.

[39] S. Rezvani, Ranking method of trapezoidal intuitionistic fuzzy numbers, Annals of Fuzzy Mathematics and Informatics 5(3) (2013) 515-523.

[40] S. Das, D. Guha, Ranking of intuitionistic fuzzy number by centroid point, Journal of Industrial and Intelligent Information 1(2) (2013) 107-110.

[41] Z. Zhang, Z. Xu, The orders of intuitionistic fuzzy numbers, Journal of Intelligent and Fuzzy Systems: Applications in Engineering and Technology 28 (2015) 505-511.

[42] D.H. Hong, C.H. Choi, Multi-criteria fuzzy decision making problems based on vague set theory, Fuzzy Sets and Systems 114 (2000) 103-113.

[43] E. Szmidt, J. Kacprzyk, Distances between intuitionistic fuzzy sets, Fuzzy Sets and Systems 114 (2000) 505-518.

[44] D.F. Li, Compromise ratio method for fuzzy multi-attribute group decision making, Applied Soft Computing 7(3) (2007) 807-817.

[45] A. Kelemenis, D. Askounis, A new TOPSIS-based multi-criteria approach to personnel selection, Expert Systems with Applications 37(7) (2010) 4999-5008.

[46] S. Opricovic, G.H. Tzeng, Compromise solution by multi-criteria decision making methods: A comparative analysis of VIKOR and TOPSIS, European Journal of Operational Research 156(2) (2004) 445-455.

[47] Y.M. Wang, Y. Luo, Area ranking of fuzzy numbers based on positive and negative ideal points, Computers and Mathematics with Applications 58(9) (2009) 1769-1779.

[48] D. Yong, L. Qi, A TOPSIS-based centroid-index ranking method of fuzzy numbers and its application in decision-making, Cybernetics and Systems: An International Journal 36(6) (2005) 581-595.

[49] H. Deng, Comparing and ranking fuzzy numbers using ideal solutions, Applied Mathematical Modelling 38(6) (2014) 1638-1646.

[50] M. Delgado, M. Vila, W. Voxman, On a canonical representation of fuzzy numbers, Fuzzy Sets and Systems 93(1) (1998) 125-135.

[51] M. Delgado, M. Vila, W. Voxman, A fuzziness measure for fuzzy numbers: Applications, Fuzzy Sets and Systems 94(2) (1998) 205-216.

[52] L. Anzilli, G. Facchinetti, Ambiguity of fuzzy quantities and a new proposal for their ranking, Przeglad Elektrotechniczny-Electrical Review 10 (2012) 280-283.

[53] M. Jiménez, M. Arenas, A. Bilbao, M.V. Rodíguez, Solving a fuzzy multiobjective linear programming problem through the value and the ambiguity of fuzzy numbers, in: Multiobjective Programming and Goal Programming, Springer, 2009, pp.187-195.

[54] S. Geetha, V. Nayagam, R. Ponalagusamy, A complete ranking of incomplete interval information, Expert Systems with Applications 41(4) (2014) 1947-1954.

[55] J.Q. Wang, J.J. Li, Multi-criteria fuzzy decision-making method based on cross entropy and score functions, Expert Systems with Applications 38(1) (2011) 1032-1038.

[56] H.W. Liu, G.J. Wang, Multi-criteria decision-making methods based on intuitionistic fuzzy sets, Eu-

ropean Journal of Operational Research 179(1) (2007) 220-233.

[57] Z.S. Xu, Intuitionistic preference relations and their application in group decision making, Information Sciences 177(11) (2007) 2363-2379.

[58] D.F. Li, The GOWA operator based approach to multiattribute decision making using intuitionistic fuzzy sets, Mathematical and Computer Modelling 53(5) (2011) 1182-1196.

[59] R.R. Yager, On a general class of fuzzy connectives, Fuzzy Sets and Systems 4(3) (1980) 235-242.

[60] S. Murakami, H. Maeda, S. Imamura, Fuzzy decision analysis in the development of centralized regional energy control systems, Energy Developments in Japan 6(4) (1984) 379-396.

[61] C.H. Cheng, A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems 95(3) (1998) 307-317.

[62] T.C. Chu, C.T. Tsao, Ranking fuzzy numbers with an area between the centroid point and original point, Computers and Mathematics with Applications 43(1) (2002) 111-117.

[63] Y.M. Wang, J.B. Yang, D.L. Xu, K.S. Chin, On the centroids of fuzzy numbers, Fuzzy Sets and Systems 157(7) (2006) 919-926.

[64] L.Q. Dat, V.F. Yu, S.Y. Chou, An improved ranking method for fuzzy numbers based on the centroid-index, International Journal of Fuzzy Systems 14(3) (2012) 413-419.

[65] N.N. Karnik, J.M. Mendel, Centroid of a type-2 fuzzy set, Information Sciences 132 (2001) 195-220.

[66] J.T. Starczewski, Centroid of triangular and Gaussian type-2 fuzzy sets, Information Sciences 280 (2014) 289-306.

[67] S.H. Cha, Comprehensive survey on distance/similarity measures between probability density functions, International Journal of Mathematical Models and Methods in Applied Sciences 1(2) (2007) 300-307.

[68] X. Wang, E.E. Kerre, Reasonable properties for the ordering of fuzzy quantities (I), Fuzzy Sets and Systems 118(3) (2001) 375-385.

[69] Z.S. Xu, Intuitionistic fuzzy aggregation operators, IEEE Transactions on Fuzzy Systems 15(6) (2007) 1179-1187.

[70] M. Casanovas, J.M. Merigo, Fuzzy aggregation operators in decision making with Dempster-Shafer belief structure, Expert Systems with Applications 39 (2012) 7138-7149.